Math 136 (Calculus for life sciences)

15.1 ~ 15.2 Limit of a function

Ex 1 $\quad f(x) = x^2$

$f(2.1) = 4.41$

$f(2.01) = 4.0401$

$f(2.001) = 4.004001$

$f(1.9) = 3.61$

$f(1.99) = 3.9601$

$f(1.999) = 3.996001$

As x approaches 2

$f(x)$ approaches 4.

natural, since $f(2) = 4$.

Ex 2

$$f(x) = \frac{x^2 - 1}{x - 1}$$ "a = 1" "L = 2"

$$f(1.1) = \frac{1.21 - 1}{1.1 - 1} = 2.1$$

$$f(1.01) = \frac{1.0201 - 1}{1.01 - 1} = 2.01$$

$$f(0.9) = 1.9$$

$$f(0.99) = 1.99$$

As x approaches 1

$f(x)$ approaches 2.

$f(1)$ is not defined.

$$f(x) = \frac{x^2 - 1}{x - 1} = \frac{(x+1)(x-1)}{x-1}$$

$$= x + 1$$ ← defined for $x = 1$

Ex 3 $\quad f(x) = \dfrac{\sin x}{x}$

$f(0.1) = 0.998334\ldots$

$f(0.01) = 0.999983\ldots$

As x approaches 0,

$f(x)$ approaches 1.

__Def__ We say the __limit__ of $f(x)$, as x approaches a, is L, if we can make the value of $f(x)$ arbitrarily close to L by taking x sufficiently close but not equal to a.

$$\lim_{x \to a} f(x) = L$$

$$\lim_{x \to 2} x^2 = 4$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

- For "elementary functions" (polynomials, fractions, trig., exp., log.), if you plug in $x=a$ and it makes sense, then it's the limit.
- Sometimes we need to first simplify.

· Properties of limit:

Suppose $\lim\limits_{x \to a} f(x) = L$,

$\lim\limits_{x \to a} g(x) = M$. then

$$\lim_{x \to a} \left(f(x) \pm g(x) \right) = L \pm M$$

$$\lim_{x \to a} \left(f(x) g(x) \right) = L \cdot M$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M} \quad (M \neq 0)$$

Ex 4 Compute

$$\lim_{x \to 2} \frac{x-2}{x^2+2x-8}$$

$$= \lim_{x \to 2} \frac{x-2}{(x+4)(x-2)}$$

$$= \lim_{x \to 2} \frac{1}{x+4} = \frac{1}{6}$$

$$\lim_{x \to -3} \frac{x+3}{x^2+6x+10} = 0$$

$$\lim_{x \to 0} \frac{2x+3\sin x}{x}$$

$$= \lim_{x \to 0} \left(2 + 3 \cdot \frac{\sin x}{x} \right)$$

$$= 2 + 3 \cdot 1 = 5$$

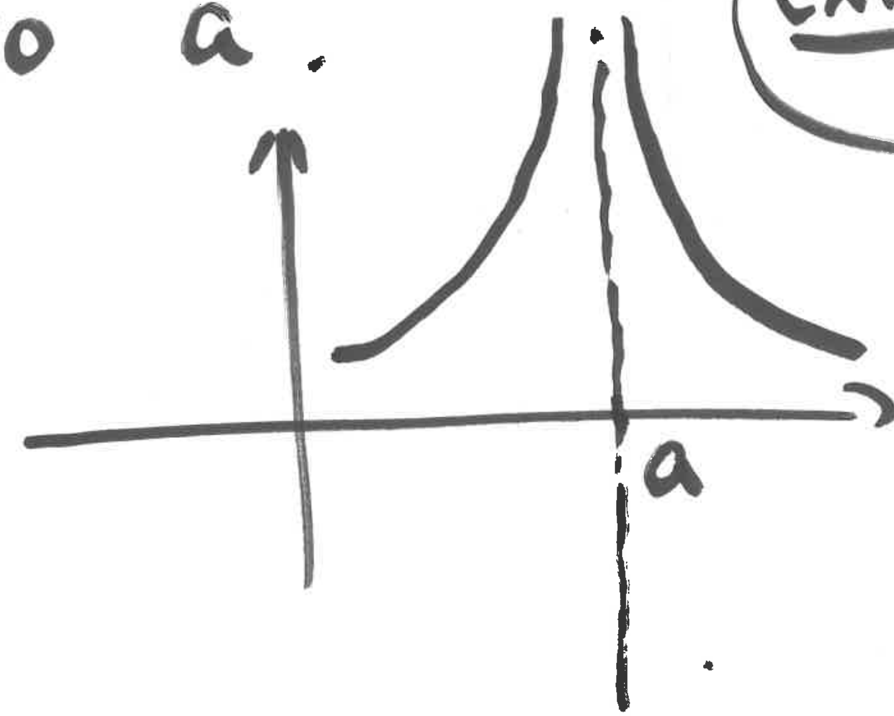
Def $\lim_{x \to a} f(x) = \infty$ means

$f(x)$ gets arbitrarily large

if x gets arbitrarily close

to a.

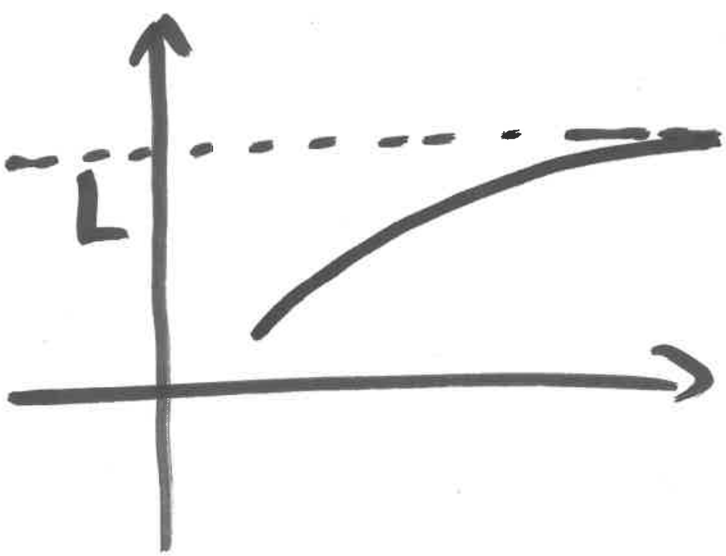
ex. $\lim_{x \to 0} \frac{1}{x^2} = \infty$



Def $\lim_{x \to \infty} f(x) = L$ means

$f(x)$ gets very close to L

if x gets arbitrarily large



$$\boxed{\underline{\text{ex.}}\ \lim_{x \to \infty} \frac{1}{x} = 0}$$

Ex 5

$$\lim_{x\to\infty} \frac{x}{2x+1} = \lim_{x\to\infty} \frac{1}{2+\frac{1}{x}}$$

$$= \frac{1}{2+0} = \frac{1}{2}.$$

$$\lim_{x \to -\infty} \frac{x^2+1}{x-2} = \lim_{x \to -\infty} \frac{x + \frac{1}{x}}{1 - \frac{2}{x}}$$

$$= -\infty.$$

" $\lim_{x \to \infty} \frac{\text{poly.}}{\text{poly.}}$ " : divide by the smaller highest-power.

$$\lim_{x\to\infty}\left(\sqrt{x+2}-\sqrt{x}\right)$$

$$=\lim_{x\to\infty}\frac{(x+2)-x}{\sqrt{x+2}+\sqrt{x}}=0$$

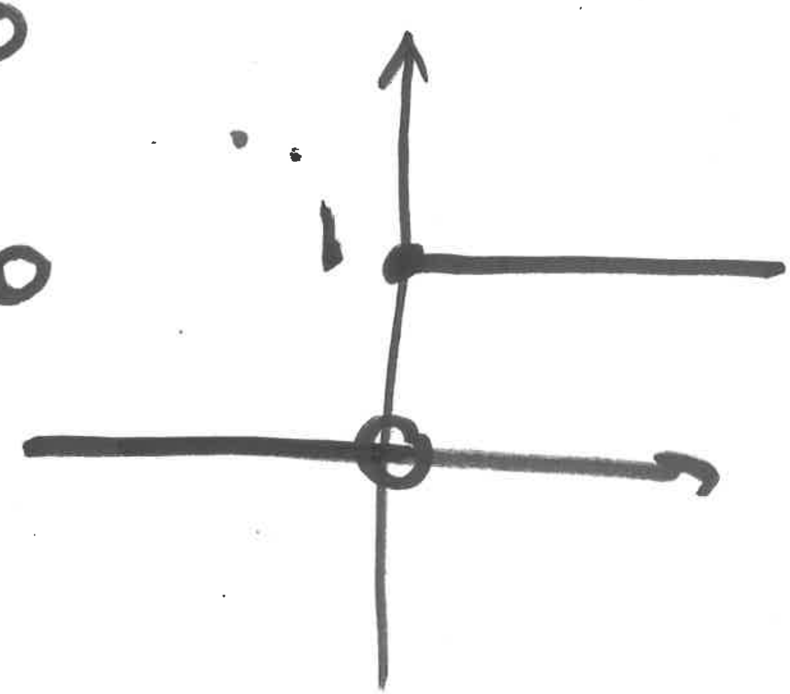
$$a^2 - b^2 = (a+b)(a-b)$$

$$a - b = \frac{a^2 - b^2}{a+b}$$

Sometimes $\lim_{x \to a} f(x)$

does not exist (DNE)

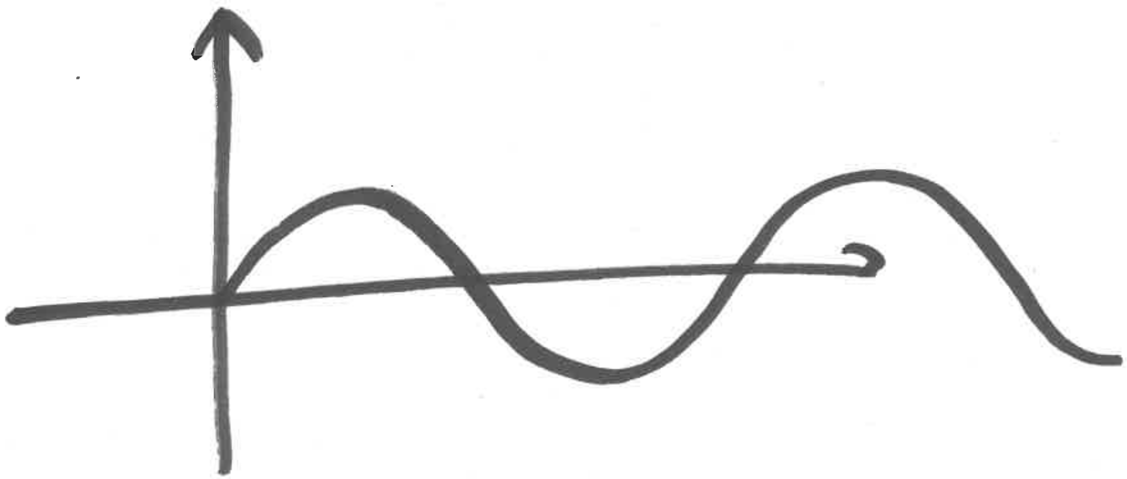
$$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$



$\lim_{x \to 0} f(x)$ DNE.

$f(0.01) = 1 \qquad f(-0.01) = 0$

$$\lim_{x\to\infty} \sin x \quad \text{DNE}$$



$$\lim_{x\to\infty} \frac{\sin x}{x} = 0$$