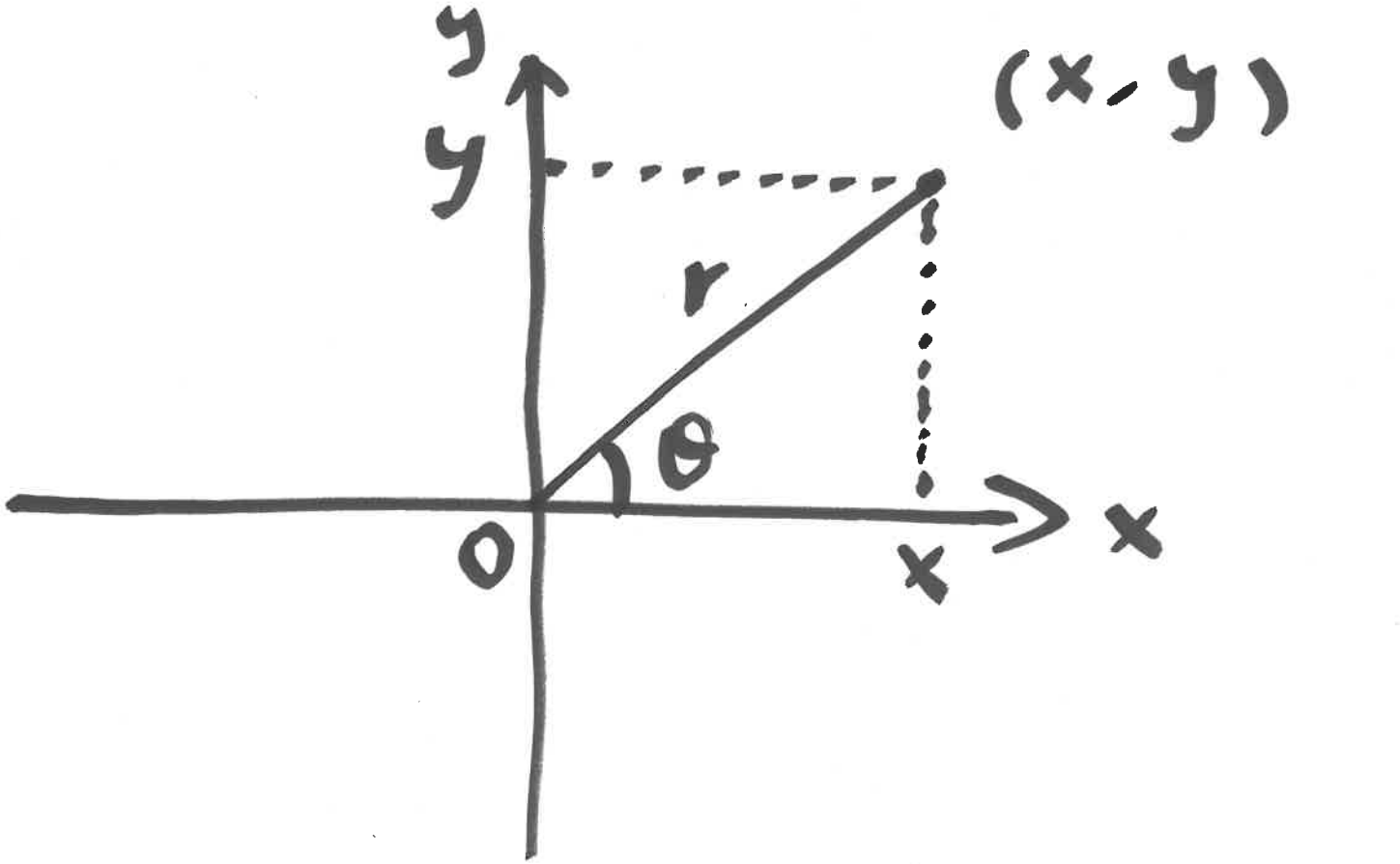
10.1 Polar Coordinates



Cartesian ⟷ Polar

(x, y) ⟷ (r, θ)

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \qquad r \geq 0$$

$$r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

θ is not unique!

ex 1 Given polar coord.,

Compute cartesian coord

$(2, -\frac{\pi}{3})$, $(1, \pi)$

$r=2$, $\theta = -\frac{\pi}{3}$

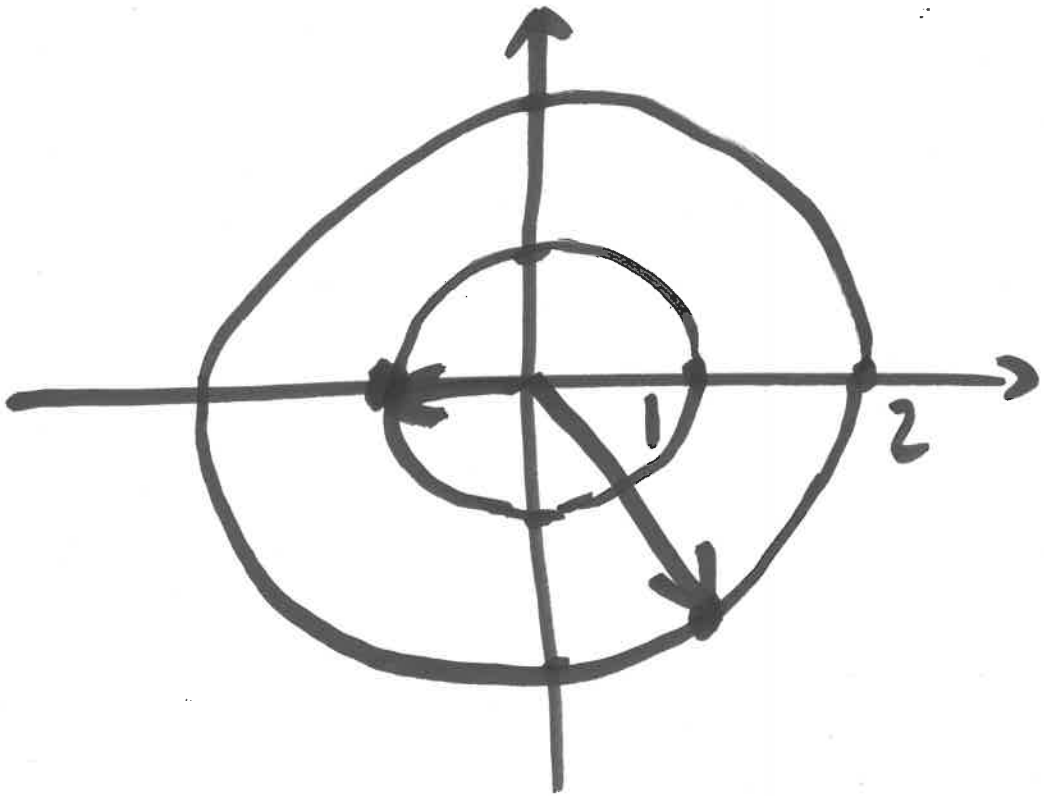
$x = 2\cos(-\frac{\pi}{3}) = 2 \cdot \frac{1}{2} = 1$

$y = 2\sin(-\frac{\pi}{3}) = 2 \cdot (-\frac{\sqrt{3}}{2}) = -\sqrt{3}$

$r = 1, \theta = \pi$

$x = 1 \cdot \cos \pi = -1$

$y = 1 \cdot \sin \pi = 0$



ex Given cartesian coord, find **all** polar coord.

$(-1, 1)$

$x = -1, \quad y = 1$

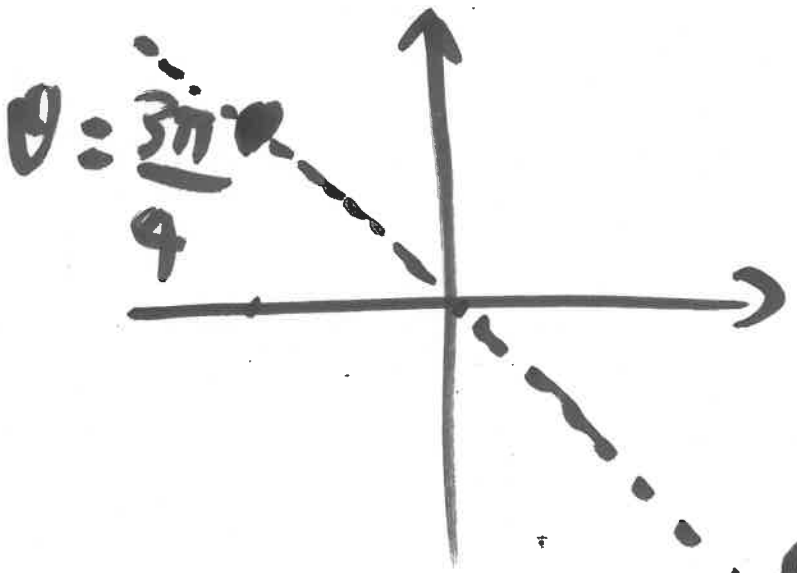
$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\tan\theta = \frac{1}{-1} = -1$

one choice is

$$\theta = \tan^{-1}(-1) = -\tan^{-1} 1 = -\frac{\pi}{4}$$

not correct direction



$\Rightarrow \pm\pi$

$\theta = \frac{3\pi}{4}$

all solutions:

$$\theta = \frac{3\pi}{4} + 2n\pi$$

n integer

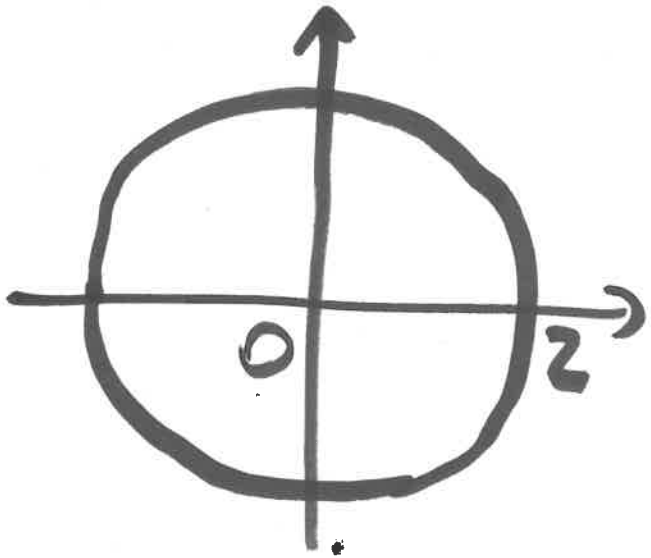
ex 3 Write cartesian eqs into polar.

① $x^2 + y^2 = 4$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 4$

$r^2 = 4$

$r = 2$



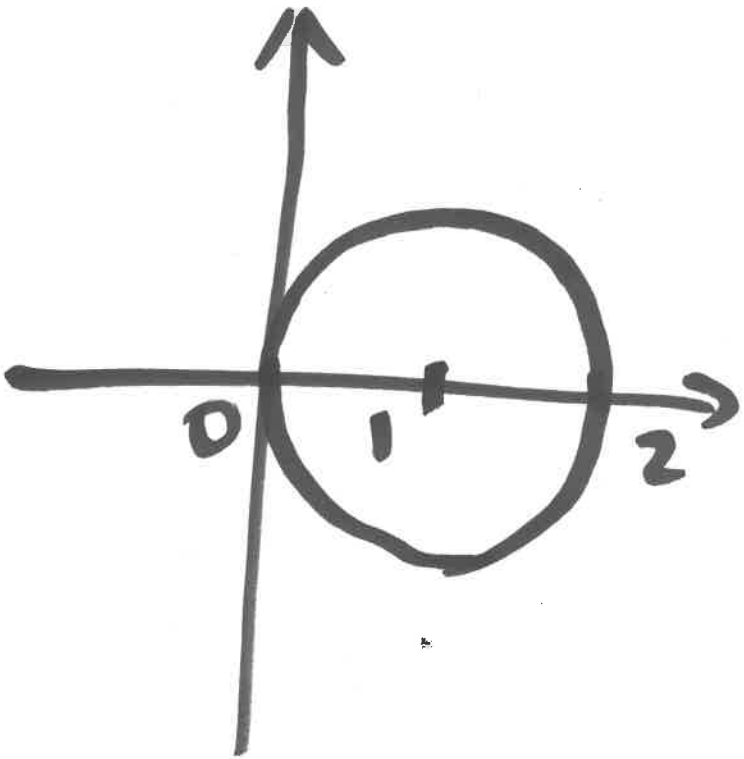
② $(x-1)^2 + y^2 = 1$

$x^2 - 2x + 1 + y^2 = 1$

$x^2 - 2x + y^2 = 0$

$r^2 - 2r\cos\theta = 0$

$r = 2\cos\theta$.



Ex 4 Write polar eqs into cartesian

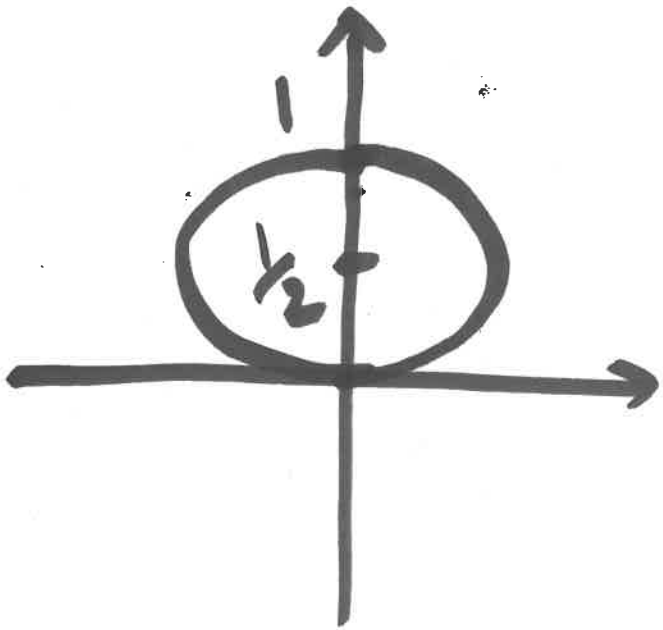
① $r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$

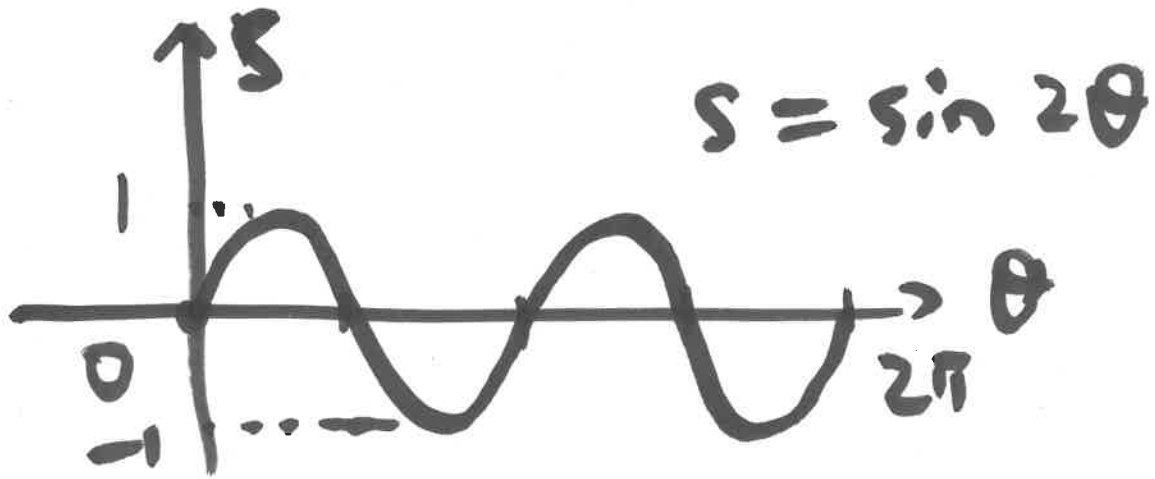


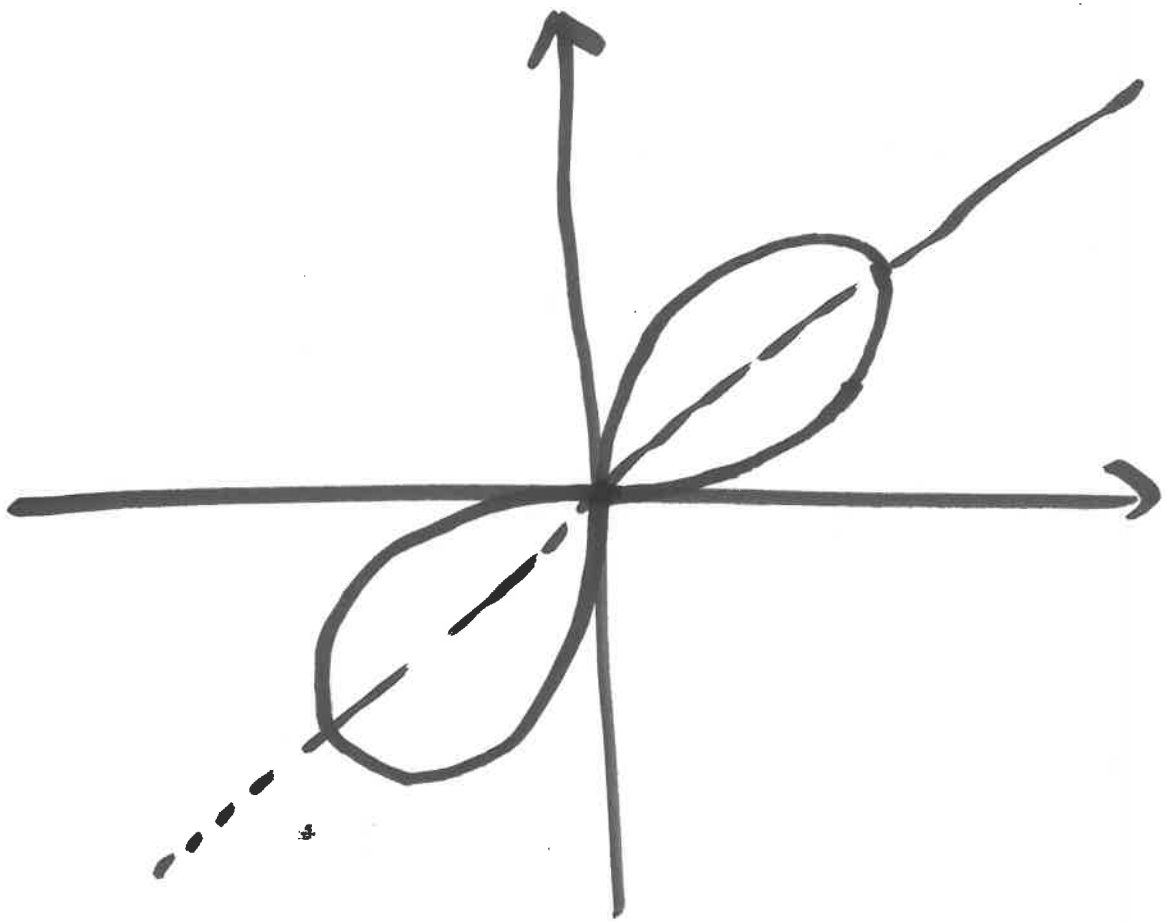
② $r^2 = \sin 2\theta$

$r^2 = 2 \sin\theta \cos\theta$

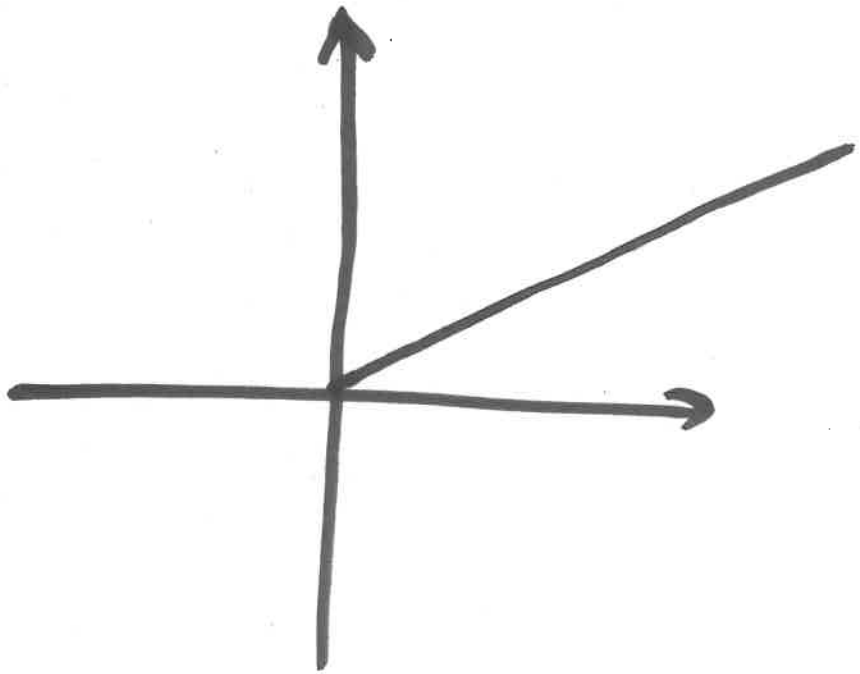
$r^4 = 2 r\sin\theta \cdot r\cos\theta$

$(x^2+y^2)^2 = 2xy$




$\theta = \frac{\pi}{4}$

(3) $\theta = \frac{\pi}{6}$



$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\frac{y}{x} = \frac{1}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}x$$

$$x \geq 0$$

④ $r = \dfrac{1}{2 - \cos\theta}$

$2r - r\cos\theta = 1$

$2\sqrt{x^2+y^2} - x = 1$

$2\sqrt{x^2+y^2} = x + 1$

$4(x^2+y^2) = x^2 + 2x + 1$

$3x^2 - 2x + 4y^2 - 1 = 0$

$$3\left(x-\frac{1}{3}\right)^2+4y^2=\frac{4}{3}$$

$$\frac{\left(x-\frac{1}{3}\right)^2}{4/9}+\frac{y^2}{1/3}=1$$