Complex numbers

A complex number is

$z = a + bi$, $a, b \in \mathbb{R}$

real part

imaginary part.

i is a symbol satisfying

$i^2 = -1$

All complex numbers: $\mathbb{C}$

$$(a+bi)+(c+di)$$

$$=(a+c)+(b+d)i$$

$$(a+bi)\cdot(c+di)$$

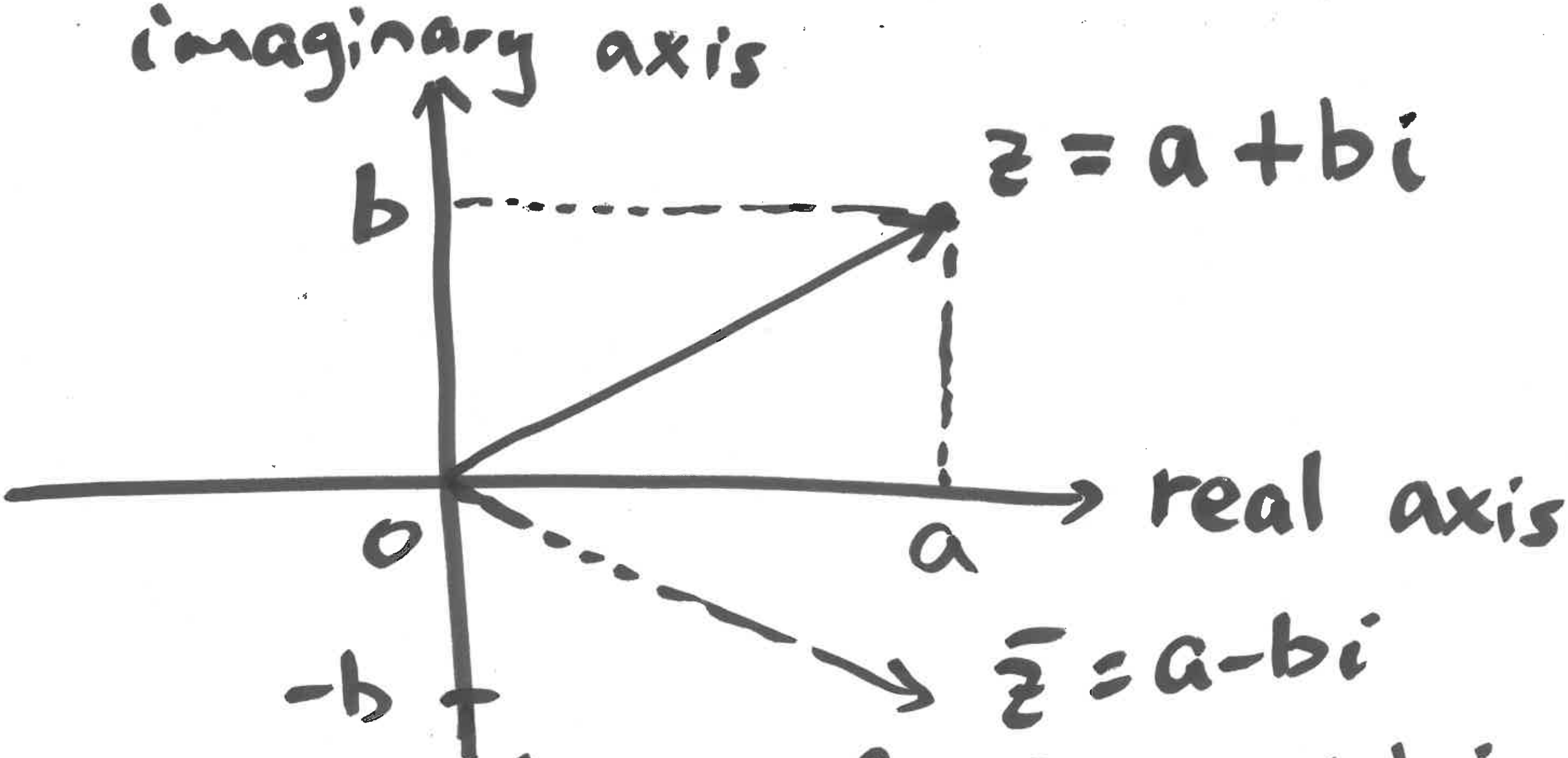
$$=a\cdot c+a\cdot di+bi\cdot c+bi\cdot di$$

$$=(ac-bd)+(ad+bc)i$$

$$\frac{1}{a+bi} = \frac{a-bi}{(a+bi)(a-bi)}$$

$$= \frac{a-bi}{a^2+b^2}$$

The complex plane



The **modulus** of $z = a + bi$

is $|z| = \sqrt{a^2 + b^2}$

The <u>distance</u> between

$z_1 = a + bi$ and $z_2 = c + di$

is $|z_1 - z_2| = \sqrt{(a-c)^2 + (b-d)^2}$

The <u>conjugate</u> of $z = a + bi$

is $\bar{z} = a - bi$

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

$$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$$|z|^2 = z \cdot \bar{z}$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

ex 1 Simplify:

$$(3+4i)(1-2i)+i$$

$$= 3 - 6i + 4i - 4i \cdot 2i \quad + i$$

$$= 11 - i$$

$$(\overline{1+i})^2 = (1-i)^2$$

$$= 1 - 2i + i^2$$

$$= -2i$$

$$\frac{i}{3-4i} = \frac{i(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{-4+3i}{25}$$

$$|(1+i)^6| = |1+i|^6$$

$$= \left(\sqrt{1^2+1^2}\right)^6$$

$$= (\sqrt{2})^6 = 8$$

ex 2 Solve equations in complex numbers:

① $z^2 = -4$

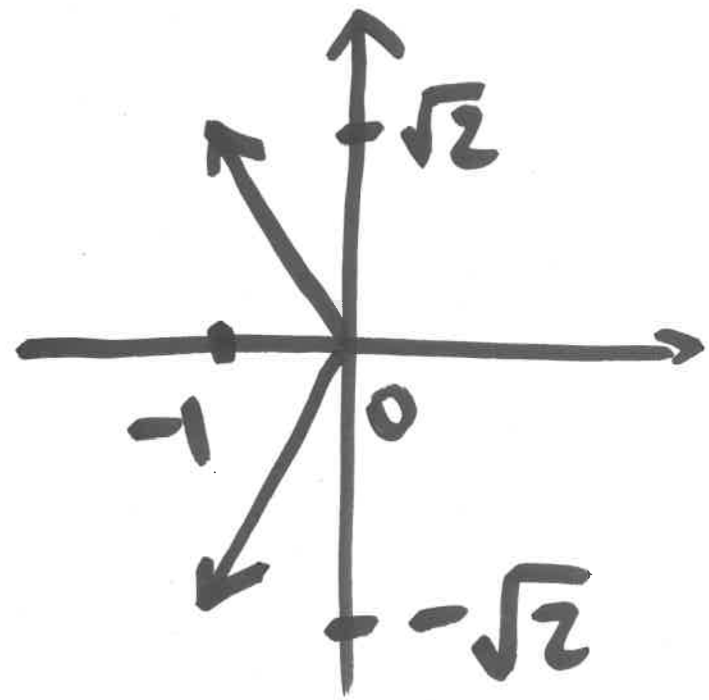
$z = \pm\sqrt{-4} = \pm 2i$

(2) $z^2 + 2z + 3 = 0$

$$z = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 3}}{2}$$

$$= \frac{-2 \pm \sqrt{-8}}{2}$$

$$= -1 \pm \sqrt{2}\, i$$



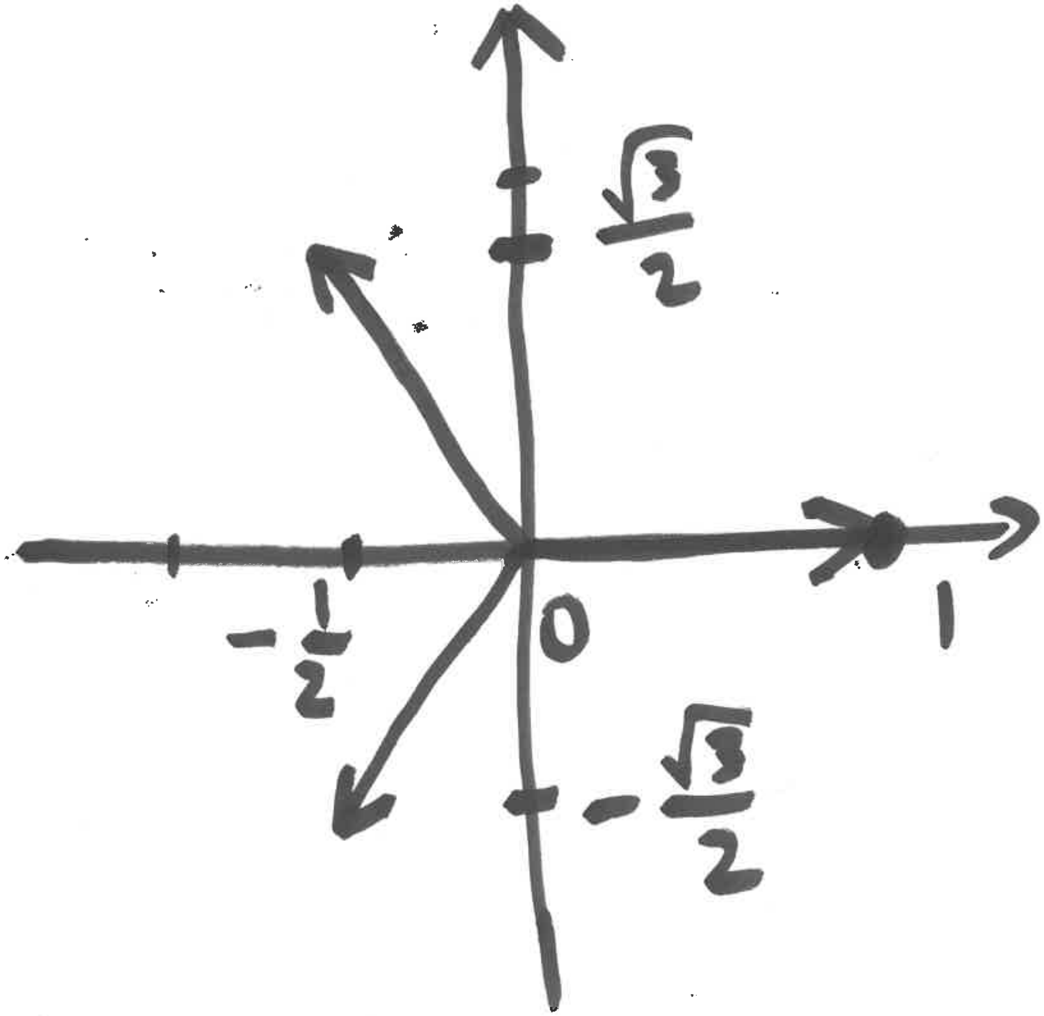
③ $z^3 - 1 = 0$

$$(z-1)(z^2+z+1) = 0$$

$$z = 1 \quad \text{or} \quad z^2 + z + 1 = 0$$

$$z = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\, i}{2}$$



$\frac{\sqrt{3}}{2}$
$-\frac{1}{2}$
0
1
$-\frac{\sqrt{3}}{2}$

Fundamental theorem of algebra

Every nonconstant polynomial $p(z)$ with coefficients in $\mathbb{C}$ has a root in $\mathbb{C}$

If $p(z)$ has degree n, then $p(z)$ can be factored as

$$p(z) = c\,(z - z_1)(z - z_2) \cdots (z - z_n)$$

$$z^3 - 1 = (z-1)\left(z - \frac{-1+\sqrt{3}\,i}{2}\right) \cdot \left(z - \frac{-1-\sqrt{3}\,i}{2}\right).$$