Chapter 8 review

ex 1 $\int x(\ln x)^2\,dx$

$u=(\ln x)^2 \qquad v=\frac{1}{2}x^2$

$du=2\cdot\ln x\cdot\frac{1}{x}dx \qquad dv=x\,dx$

$=\frac{1}{2}x^2(\ln x)^2-\int\frac{1}{2}x^2\cdot 2\ln x\cdot\frac{1}{x}dx$

$=\frac{1}{2}x^2(\ln x)^2-\int x\ln x\,dx$

$$\left(\begin{array}{ll} u = \ln x & v = \frac{1}{2}x^2 \\ du = \frac{1}{x}dx & dv = x\,dx \end{array}\right)$$

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \int \frac{1}{2}x^2 \cdot \frac{1}{x}dx$$

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C$$

ex 2

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx$$

want $4x^2 = 9\tan^2 u$

$2x = 3\tan u$

$$x = \frac{3}{2}\tan u$$

$$\left(dx = \frac{3}{2}\sec^2 u\,du \right)$$

$$= \int \frac{1}{\sqrt{9+9\tan^2 u}} \cdot \frac{3}{2}\sec^2 u\,du$$

$$= \int \frac{1}{3\sec u} \cdot \frac{3}{2}\sec^2 u \, du$$

$$= \frac{1}{2}\int \sec u \, du$$

$$= \frac{1}{2}\ln\left|\sec u + \tan u\right| + C$$

$$\left(\begin{array}{l} \tan u = \frac{2}{3}x \\ \sec u = \sqrt{1 + (\frac{2}{3}x)^2} \end{array} \right)$$

$$= \frac{1}{2}\ln\left|\sqrt{1+(\tfrac{2}{3}x)^2} + \frac{2}{3}x\right| + C$$

ex 3 $\displaystyle\int \frac{x^2}{(x-1)^2(x+2)}\,dx$

$$= \int \left(\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2} \right) dx$$

$$x^2 = A\cdot(x-1)(x+2) + B(x+2)$$
$$+ C(x-1)^2$$

$x = 1$: $1 = B \cdot (1+2)$

$$B = \frac{1}{3}$$

$x = -2$: $4 = C \cdot (-2-1)^2$

$$C = \frac{4}{9}$$

Constant : $0 = -2A + 2B + C$

$$2A = 2B + C$$

$$A = B + \frac{1}{2}C = \frac{1}{3} + \frac{1}{2} \cdot \frac{4}{9}$$

$$A = \frac{5}{9}$$

$$\int \frac{x^2}{(x-1)^2(x+2)}\,dx$$

$$= \int \left(\frac{5}{9} \cdot \frac{1}{x-1} + \frac{1}{3}\frac{1}{(x-1)^2} + \frac{4}{9} \cdot \frac{1}{x+2} \right) dx$$

$$= \frac{5}{9}\ln|x-1| - \frac{1}{3}\cdot\frac{1}{x-1} + \frac{4}{9}\ln|x+2| + C$$

<u>ex 4</u> Approximate $\int_0^2 \frac{1}{1+x}\,dx$

by Simpson and estimate error $(n=4)$

k	0	1	2	3	4
x_k	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x_k)$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

$$\int_0^2 \frac{1}{1+x}\,dx \approx \frac{1}{3}\cdot\frac{1}{2}\Big[1\cdot 1 + 4\cdot\frac{2}{3}$$

$$+ 2\cdot\frac{1}{2} + 4\cdot\frac{2}{5} + 1\cdot\frac{1}{3}\Big].$$

$$E_4^S \le \frac{\max f^{(4)}(x)}{180\cdot 4^4}\cdot(2-0)^5$$

$$f^{(4)}(x) = (-1)\cdot(-2)\cdot(-3)\cdot(-4)\cdot\frac{1}{(1+x)^5}$$

$$= \frac{24}{(1+x)^5}$$

for $0 \le x \le 2$,

max of $\dfrac{24}{(1+x)^5}$ is 24

(at $x=0$).

$$E_4^S \le \frac{24}{180 \cdot 4^4} \cdot 2^5$$

$\underline{ex\ 5}$ $\displaystyle\int_0^{\boxed{\infty}} \frac{1}{1+\sqrt{x}}\,dx$ conv/div ?

$$= \lim_{c\to\infty} \int_0^c \frac{1}{1+\sqrt{x}}\,dx$$

$$\int \frac{1}{1+\sqrt{x}}\,dx = \int \frac{1}{1+u}\cdot 2u\,du$$

$$\left(\begin{array}{c} x = u^2 \\ dx = 2u\,du \end{array}\right)$$

$$= 2 \int \frac{u}{1+u}\, du$$

$$= 2 \int \frac{(1+u)-1}{1+u}\, du$$

$$= 2 \int \left(1 - \frac{1}{1+u}\right) du$$

$$= 2\left(u - \ln|1+u|\right) + C$$

$$= 2\left(\sqrt{x} - \ln|1+\sqrt{x}|\right) + C$$

$$\lim_{c\to\infty} \int_0^c \frac{1}{1+\sqrt{x}}\,dx$$

$$= \lim_{c\to\infty} 2\left(\sqrt{x} - \ln\left|1+\sqrt{x}\right|\right)\Big|_0^c$$

$$= \lim_{c\to\infty} 2\left(\sqrt{c} - \ln(1+c)\right)$$

goes to ∞ faster

$$= \infty$$

$\boxed{\text{div}}$