Chapter 7 review

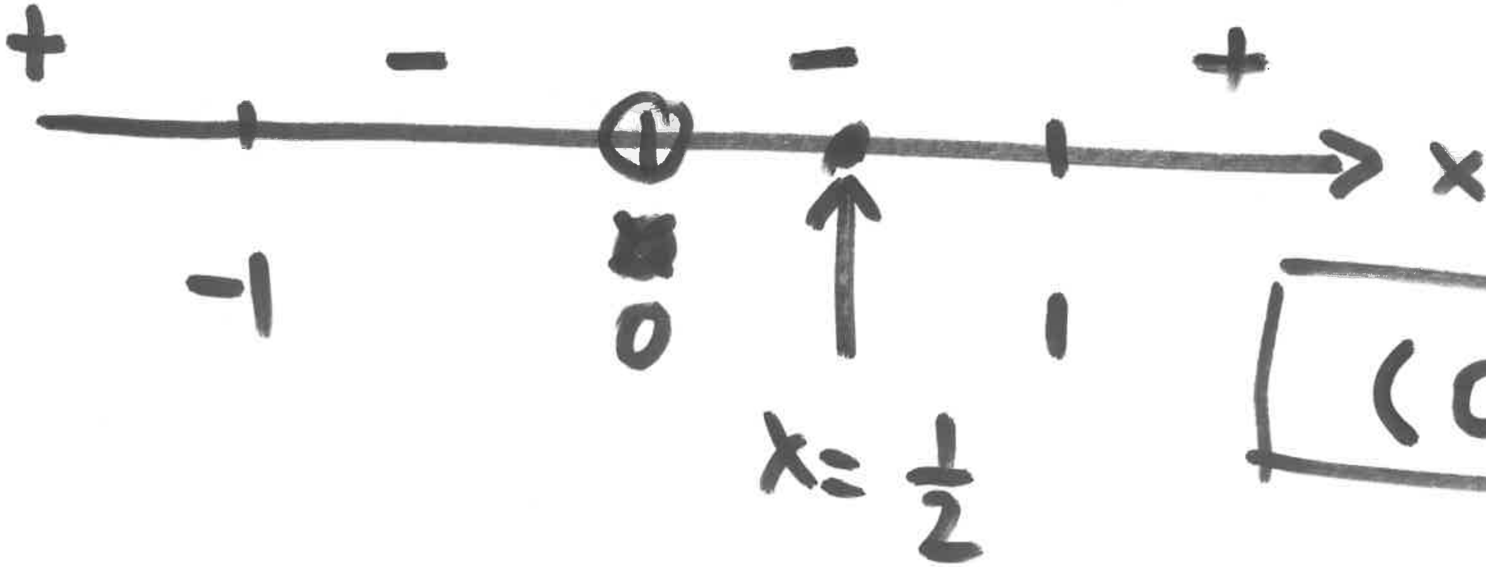
ex 1 $f(x) = \frac{1}{x} + x$. Find the largest interval containing $x = \frac{1}{2}$ such that f has an inverse. Find this inverse.

$x \neq 0$

$$f'(x) = -\frac{1}{x^2} + 1$$

$$f'(x) = 0 \Rightarrow -1 + x^2 = 0$$

$$x^2 = 1 \qquad x = \pm 1$$



$$\boxed{(0,\ 1]}$$

$$y = \frac{1}{x} + x \qquad x \in (0, 1]$$

$$xy = 1 + x^2$$

$$x^2 - y \cdot x + 1 = 0$$

$$x = \frac{y \pm \sqrt{y^2 - 4}}{2} = \frac{y - \sqrt{y^2 - 4}}{2}$$

try $y = 3$ $\quad x = \frac{3 \pm \sqrt{5}}{2}$ want

$$f^{-1}(x) = \frac{x - \sqrt{x^2 - 4}}{2}$$

ex 2 $\left((\sin x)^{\cos x} \right)'$

$$= \left(e^{\ln\left((\sin x)^{\cos x} \right)} \right)'$$

$$= \left(e^{\cos x \cdot \ln \sin x} \right)'$$

$$= e^{\cos x \cdot \ln \sin x} \cdot \left(-\sin x \cdot \ln \sin x + \cos x \cdot \frac{1}{\sin x} \cdot \cos x \right)$$

$$\int \frac{1}{1+e^x}\,dx = \int \frac{1}{1+u}\cdot\frac{1}{u}\,du$$

$$\left(\begin{aligned} &u = e^x \\ &du = e^x\,dx = u\,dx \\ &dx = \frac{du}{u} \end{aligned}\right)$$

$$= \int \left(\frac{1}{u} - \frac{1}{1+u} \right) du$$

$$= \ln|u| - \ln|1+u| + C$$

$$= \ln e^x - \ln(1+e^x) + C$$

$$= x - \ln(1+e^x) + C$$

$$\int \frac{1}{1+e^x}\,dx = -\int \frac{1}{1+\frac{1}{u}} \cdot \frac{1}{u}\,du$$

$$\left(\begin{aligned} u &= e^{-x} \\ du &= -e^{-x}\,dx = -u\,dx \\ dx &= -\frac{du}{u} \end{aligned} \right)$$

$$= -\int \frac{1}{u+1}\,du$$

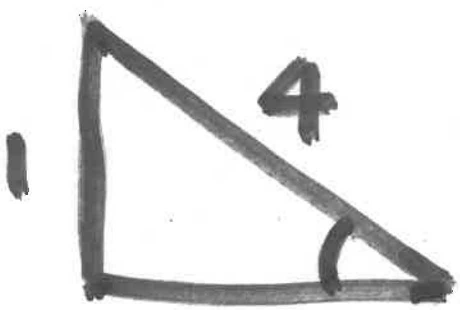
$$= -\ln|u+1| + C$$

$$= -\ln(e^{-x}+1) + C$$

ex 3 $\tan\left(\sin^{-1}\left(-\frac{1}{4}\right)\right) = ?$

$$= \tan\left(-\sin^{-1}\left(\frac{1}{4}\right)\right)$$

$$= -\tan\left(\underbrace{\sin^{-1}\left(\frac{1}{4}\right)}\right) = -\frac{1}{\sqrt{15}}$$



$$\sqrt{4^2 - 1^2} = \sqrt{15}$$

$$\int \frac{x}{\sqrt{-x^4-2x^2+3}}\,dx$$

$$\left(\begin{array}{c} u = x^2 \\ du = 2x\,dx \end{array}\right)$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{-u^2-2u+3}}\,du$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{-(u+1)^2 + 4}} \, du$$

$$= \frac{1}{2} \sin^{-1}\left(\frac{u+1}{2}\right) + C$$

$$= \frac{1}{2} \sin^{-1}\left(\frac{x^2+1}{2}\right) + C.$$

(recall $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$)

__ex 4__ $\lim\limits_{x\to\infty} x^{\frac{1}{\sqrt{x}}}$

$$= \lim_{x\to\infty} e^{\frac{1}{\sqrt{x}}\ln x}$$

$$= e^{\lim\limits_{x\to\infty} \frac{\ln x}{\sqrt{x}}}$$

"$\frac{\infty}{\infty}$"

$$= e^{\lim\limits_{x\to\infty} \frac{1/x}{\frac{1}{2}\cdot 1/\sqrt{x}}} = e^0 = 1.$$

$$= \frac{1}{1/2\cdot\sqrt{x}} \to 0$$

$$\lim_{x \to 1} \tan\left(\frac{\pi}{2}x\right) \ln x$$

$$= \lim_{x \to 1} \frac{\ln x}{\cot\left(\frac{\pi}{2}x\right)}$$

$$\underset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x \to 1} \frac{1/x}{-\frac{1}{\sin^2\left(\frac{\pi}{2}x\right)} \cdot \frac{\pi}{2}} = -\frac{2}{\pi}.$$