ex 1 $\frac{1}{2}\int 2x^3 e^{x^2}\,dx$

$$= \frac{1}{2}\int y e^y\,dy \qquad \left(\begin{array}{l} y = x^2 \\ dy = 2x\,dx \end{array}\right)$$

$$= \frac{1}{2}\left(y e^y - \int e^y\,dy\right) \qquad \left(\begin{array}{ll} u = y & v = e^y \\ du = dy & dv = e^y\,dy \end{array}\right)$$

$$= \frac{1}{2} y e^y - \frac{1}{2} e^y + C$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$$

$$\int \cos\sqrt{t}\,dt$$

$$= \int \cos x \cdot 2x\,dx$$

$$\left(\begin{aligned} x &= \sqrt{t} \\ dx &= \frac{1}{2\sqrt{t}}\,dt \\ dt &= 2\sqrt{t}\,dx \\ &= 2x\,dx \end{aligned} \right)$$

$$= 2\int x\cos x\,dx$$

$$= 2\Big(x\sin x$$

$$-\int \sin x\,dx\Big)$$

$$\left(\begin{aligned} u &= x & v &= \sin x \\ du &= dx & dv &= \cos x\,dx \end{aligned} \right)$$

$$= 2x\sin x - 2\cdot(-\cos x) + C$$

$$= 2x\sin x + 2\cos x + C$$

$$= 2\sqrt{t}\sin\sqrt{t} + 2\cos\sqrt{t} + C$$

ex 2

$$\int e^{ax} \cos bx \, dx$$

$a, b > 0$

$$= \frac{1}{a} e^{ax} \cos bx - \int \frac{1}{a} e^{ax} \cdot (-b \sin bx) \, dx$$

$u = \cos bx \qquad v = \frac{1}{a} e^{ax}$

$du = -b \sin bx \qquad dv = e^{ax} dx$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a} \int e^{ax} \sin bx \, dx$$

$$\left(\begin{array}{ll} u = \sin bx & v = \frac{1}{a} e^{ax} \\ & \\ du = b\cos bx\, dx & dv = e^{ax}\, dx \end{array} \right)$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a} \left(\frac{1}{a} e^{ax} \sin bx - \int \frac{1}{a} e^{ax}\, b\cos bx\, dx \right)$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \sin bx - \frac{b^2}{a^2} \int e^{ax} \cos bx\, dx$$

$$D = \int e^{ax} \cos bx \, dx$$

$$D = \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \sin bx$$

$$- \frac{b^2}{a^2} \cdot D$$

$$D = \frac{\frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \sin bx}{1 + \frac{b^2}{a^2}} + C.$$

$$= \frac{a e^{ax} \cos bx + b e^{ax} \sin bx}{a^2 + b^2} + C.$$

<u>ex 3</u> $f(a) = \int_0^\infty x^{a-1} e^{-x}\,dx$.

Express $f(a)$ in terms of $f(a-1)$.

$$f(1) = \int_0^\infty e^{-x}\,dx = -e^{-x}\Big|_0^\infty$$
$$= 1$$

$$f(a) = \int_0^\infty x^{a-1} e^{-x}\, dx \quad \boxed{a > 1}$$

$$\left(\begin{array}{ll} u = x^{a-1} & v = -e^{-x} \\ du = (a-1)\, x^{a-2} dx & dv = e^{-x} dx \end{array} \right)$$

$$= -x^{a-1} e^{-x} \Big|_0^\infty - \int_0^\infty (-e^{-x})$$

$$\cdot (a-1)\, x^{a-2}\, dx$$

$$= (a-1)\, f(a-1).$$

$$f(a) = (a-1)\,f(a-1)$$

$$f(1) = 1$$

$$f(2) = 1 \cdot f(1) = 1$$

$$f(3) = 2 \cdot f(2) = 2$$

$$f(4) = 3 \cdot f(3) = 6$$

$$f(5) = 4 \cdot f(4) = 24$$

$f(a) = (a-1)!$ → integer a.