exponential forms: 0^0, 1^∞, ∞^0

— put into natural exp.

take limit on exponent.

<u>ex 3</u> $\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln(x^x)}$

$= \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x}$

$= e^0 = 1$

$$\lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{1/x}$$

$$\underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = 0$$

$$\lim_{x\to\infty} x^{1/x} = \lim_{x\to\infty} e^{\ln\left(x^{1/x}\right)}$$

$$= \lim_{x\to\infty} e^{\frac{1}{x}\ln x} = e^{\lim_{x\to\infty} \frac{\ln x}{x}}$$

$$\underset{\text{"}\frac{\infty}{\infty}\text{"}}{=} e^{\lim_{x\to\infty} \frac{1/x}{1}} = e^0 = 1$$

$$\lim_{x \to 0+} (1+x)^{\frac{1}{x}} = \lim_{x \to 0+} e^{\ln\left((1+x)^{\frac{1}{x}}\right)}$$

$$= \lim_{x \to 0+} e^{\frac{1}{x}\ln(1+x)} = e^{\lim_{x \to 0+} \frac{\ln(1+x)}{x}}$$

$$\underset{\uparrow \atop \text{"}\frac{0}{0}\text{"}}{=} e^{\lim_{x \to 0+} \frac{1/(1+x)}{1}} = e^{1} = e$$

ex 4 $\lim_{x \to 0} (\csc x - \cot x)$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin x}$$

$$\underset{\text{"}\frac{0}{0}\text{"}}{\overset{\uparrow}{=}} \lim_{x \to 0} \frac{\sin x}{\cos x} = 0$$

$$\lim_{x \to \frac{\pi}{2}} (\pi^2 - 4x^2) \tan x$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\pi^2 - 4x^2}{\cot x}$$

$$\underset{\text{"}\frac{0}{0}\text{"}}{\overset{\uparrow}{=}} \lim_{x \to \frac{\pi}{2}} \frac{-8x}{-\frac{1}{\sin^2 x}} = \frac{-4\pi}{-1} = 4\pi$$

$$\lim_{x\to\infty} x^{-1/2}\ln(\ln x)$$

$$= \lim_{x\to\infty} \frac{\ln(\ln x)}{x^{1/2}}$$

$$\underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{\uparrow}{=}} \lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{\frac{1}{2}x^{-1/2}} = \lim_{x\to\infty} \frac{2x^{1/2}}{x\ln x}$$

$$= \lim_{x\to\infty} \frac{2}{x^{1/2}\ln x} = 0$$

8.1 Integration by parts

$$(uv)' = u'v + uv'$$

$$uv = \int u'v\,dx + \int uv'\,dx$$

$$\boxed{\begin{aligned} \int uv'\,dx &= uv - \int u'v\,dx \\ \int u\,dv &= uv - \int v\,du \end{aligned}}$$

ex 1 $\int x \sin x \, dx$

$$\begin{pmatrix} u = x & v = -\cos x \\ du = dx & dv = \sin x \, dx \end{pmatrix}$$

$$= x \cdot (-\cos x) - \int (-\cos x) \, dx$$

$$= -x \cos x + \sin x + C$$

$$\int x e^x \, dx = xe^x - \int e^x \, dx$$

$$= xe^x - e^x + C$$

$$\left(\begin{array}{ll} u = x & v = e^x \\ du = dx & dv = e^x \, dx \end{array} \right)$$

$$\int x \ln x \, dx = \ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$$

$$\left(\begin{array}{ll} u = \ln x & v = \frac{1}{2}x^2 \\ du = \frac{1}{x} dx & dv = x \, dx \end{array} \right)$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} \, dx$$

$$\begin{pmatrix} u = \ln x & v = x \\ du = \frac{1}{x} dx & dv = dx \end{pmatrix} \qquad = x \ln x - x + C$$

u $\quad \ln x, \ \tan^{-1} x, \ \sin^{-1} x$

polynomials $\ 1, x, x^2, \cdots$

v $\quad e^{ax}, \ \sin ax, \ \cos ax$