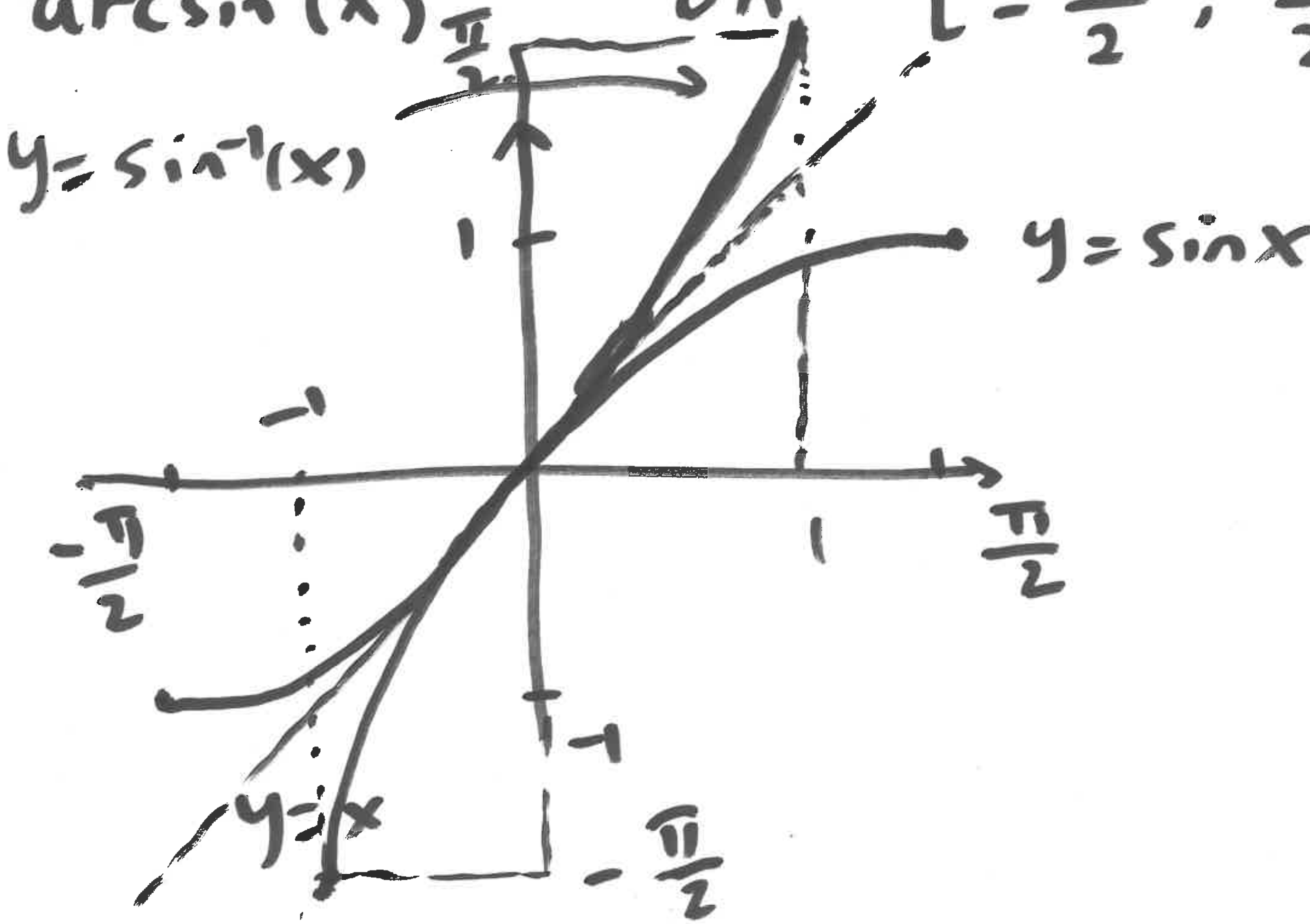
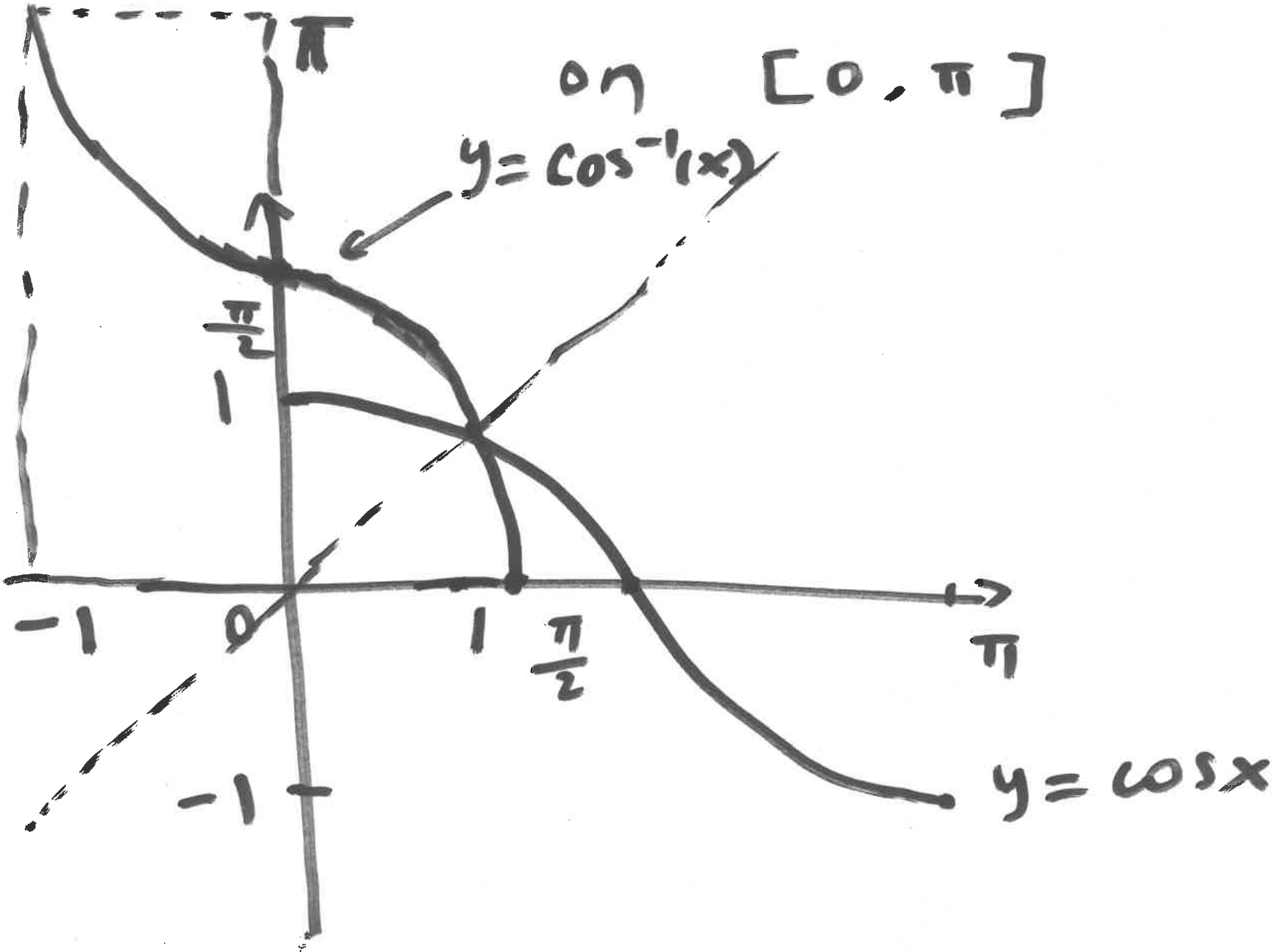
7.5 Inverse trig functions

$\sin^{-1}(x)$: inverse function of $\sin x$

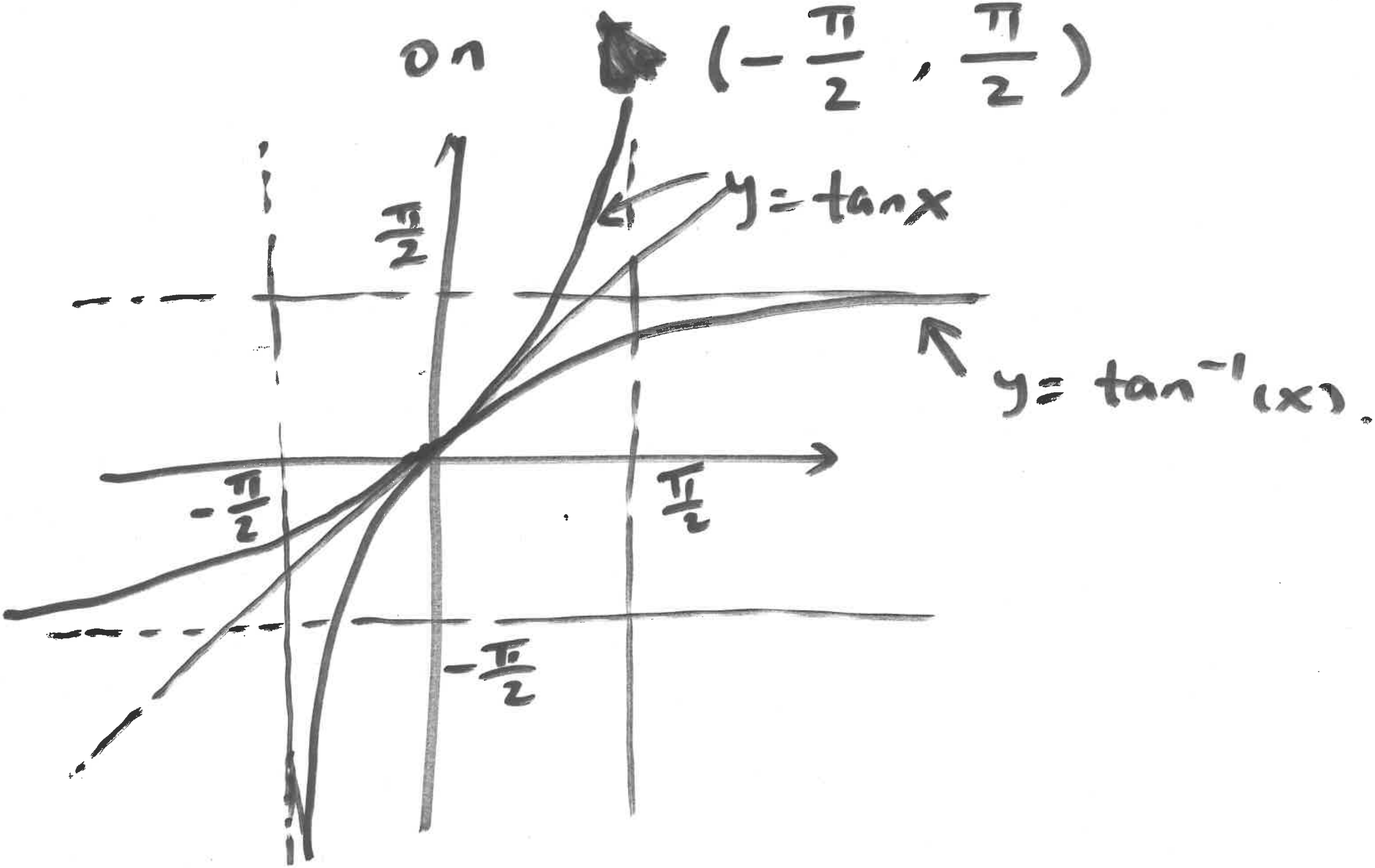
$\arcsin(x)$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$



$\cos^{-1}(x)$: inverse function of $\cos x$ on $[0, \pi]$



$\tan^{-1}(x)$: inverse function of $\tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



odd func.

	domain	range
$\sin^{-1}(x)$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}(x)$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\csc^{-1}(x)$	$(-\infty, -1] \cup [1, \infty)$	$(0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$
$\sec^{-1}(x)$	$(-\infty, -1] \cup [1, \infty)$	$[0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$
$\cot^{-1}(x)$	$(-\infty, \infty)$	$(0, \pi)$

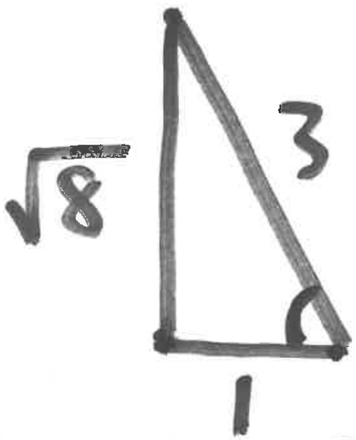
<u>ex 1</u> $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

$$\cot^{-1}(-1) \stackrel{?}{=} \tan^{-1}\left(\frac{1}{-1}\right) = \tan^{-1}(-1)$$

$$= -\tan^{-1}(1) = -\frac{\pi}{4}$$

$$\cot^{-1}(-1) = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$$

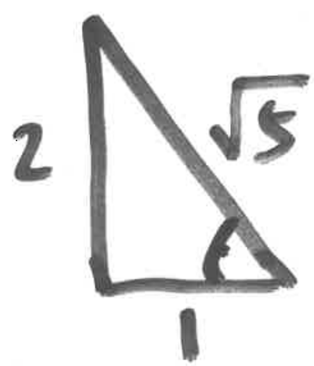
ex 2 $\tan\left(\cos^{-1}\left(\frac{1}{3}\right)\right) = \sqrt{8}$



$$\sqrt{3^2 - 1^2} = \sqrt{8}$$

$$\cos\left(\tan^{-1}(-2)\right) = \cos\left(-\tan^{-1}(2)\right)$$

$$= \cos\left(\tan^{-1}(2)\right) = \frac{1}{\sqrt{5}}$$



$$\sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

<u>ex 3</u> $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{2} + \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

- $\left(\sin^{-1}(x)\right)' = \dfrac{1}{\sqrt{1-x^2}}$

$\left(\tan^{-1}(x)\right)' = \dfrac{1}{1+x^2}$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

ex 4 $\left(e^{\sin^{-1}(x^2)}\right)'$

$$= e^{\sin^{-1}(x^2)} \left(\sin^{-1}(x^2)\right)'$$

$$= e^{\sin^{-1}(x^2)} \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$= e^{\sin^{-1}(x^2)} \frac{2x}{\sqrt{1-x^4}}$$

ex 5

$$\frac{1}{2}\int \frac{1}{\sqrt{1-4x^2}}\,2dx = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{2}\sin^{-1}(u) + C$$

$$= \frac{1}{2}\sin^{-1}(2x) + C$$

$$\left(\begin{array}{l} 4x^2 = (2x)^2 \\ u = 2x \\ du = 2\,dx \end{array}\right)$$

$$\int \frac{1}{x^2+9}\,dx = \int \frac{1}{9\left(\underbrace{\boxed{\tfrac{1}{9}x^2}}_{u^2}+1\right)}\,\boxed{dx\cdot\tfrac{1}{3}}\,3$$

$$u = \frac{1}{3}x$$

$$du = \frac{1}{3}dx$$

$$= \frac{1}{9}\cdot 3\cdot\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{3}\cdot\tan^{-1}(u) + C$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{1}{3}x\right) + C.$$