# 7.3 General exp, log

$f(x) = a^x \qquad a > 0$

$f^{-1}(x) = \log_a x$ for $a > 0, a \neq 1$

$y = a^x \iff x = \log_a y$

- Convert to natural base

$$\boxed{a^x = e^{x \ln a}} \qquad \boxed{\log_a x = \frac{\ln x}{\ln a}}$$

$$a^x = e^{\ln(a^x)} = e^{x \ln a}$$

$$\ln x = \ln\left(a^{\log_a x}\right) = \log_a x \cdot \ln a$$

- $(a^x)' = (e^{x \ln a})'$

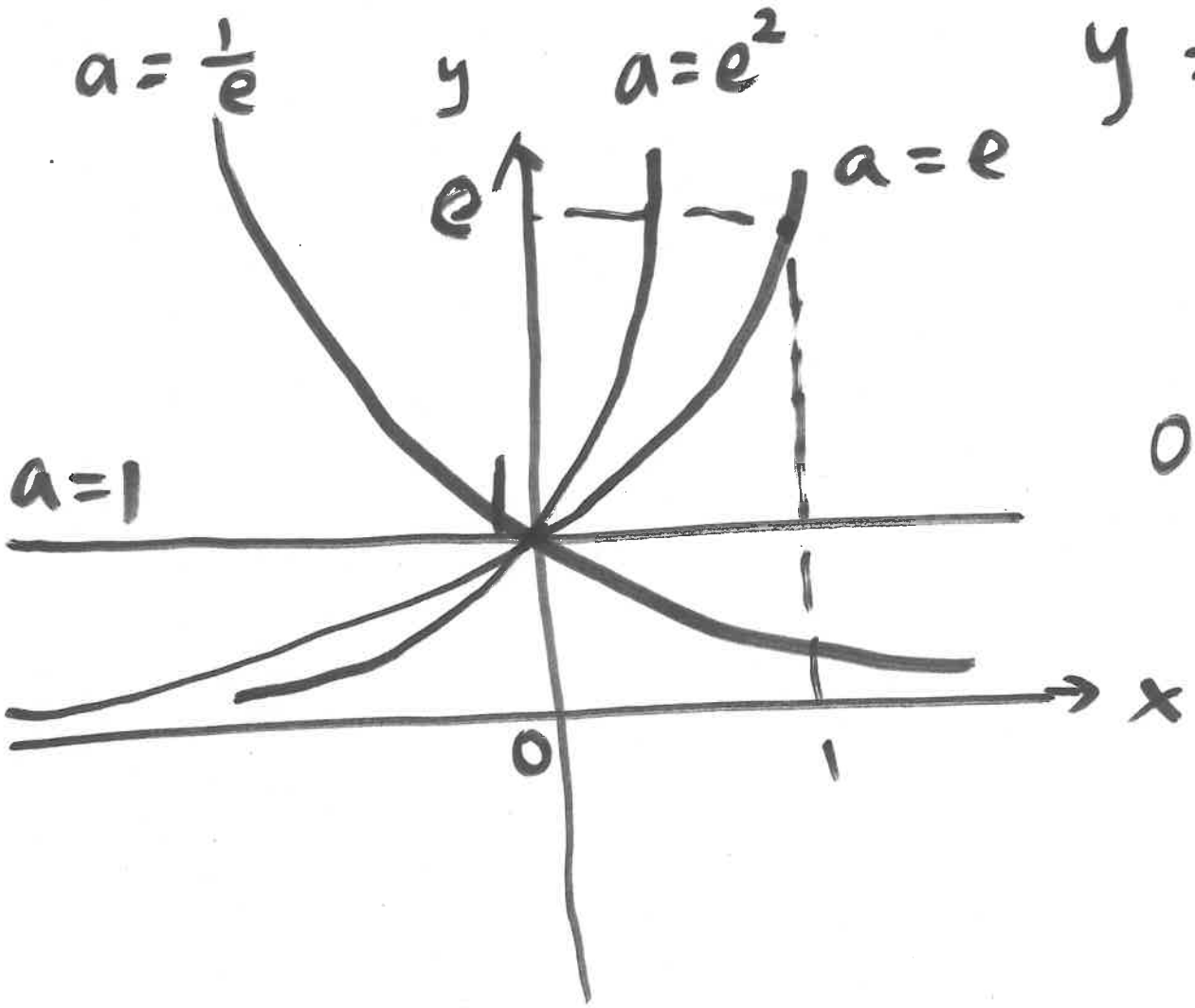
$= e^{x \ln a} \cdot \ln a = a^x \ln a$

- $(\log_a x)' = \left(\frac{\ln x}{\ln a}\right)'$

$= \frac{1}{\ln a} \cdot \frac{1}{x} = \frac{1}{x \ln a}$

- $\int a^x \, dx = \frac{a^x}{\ln a} + C \quad (a \neq 1)$

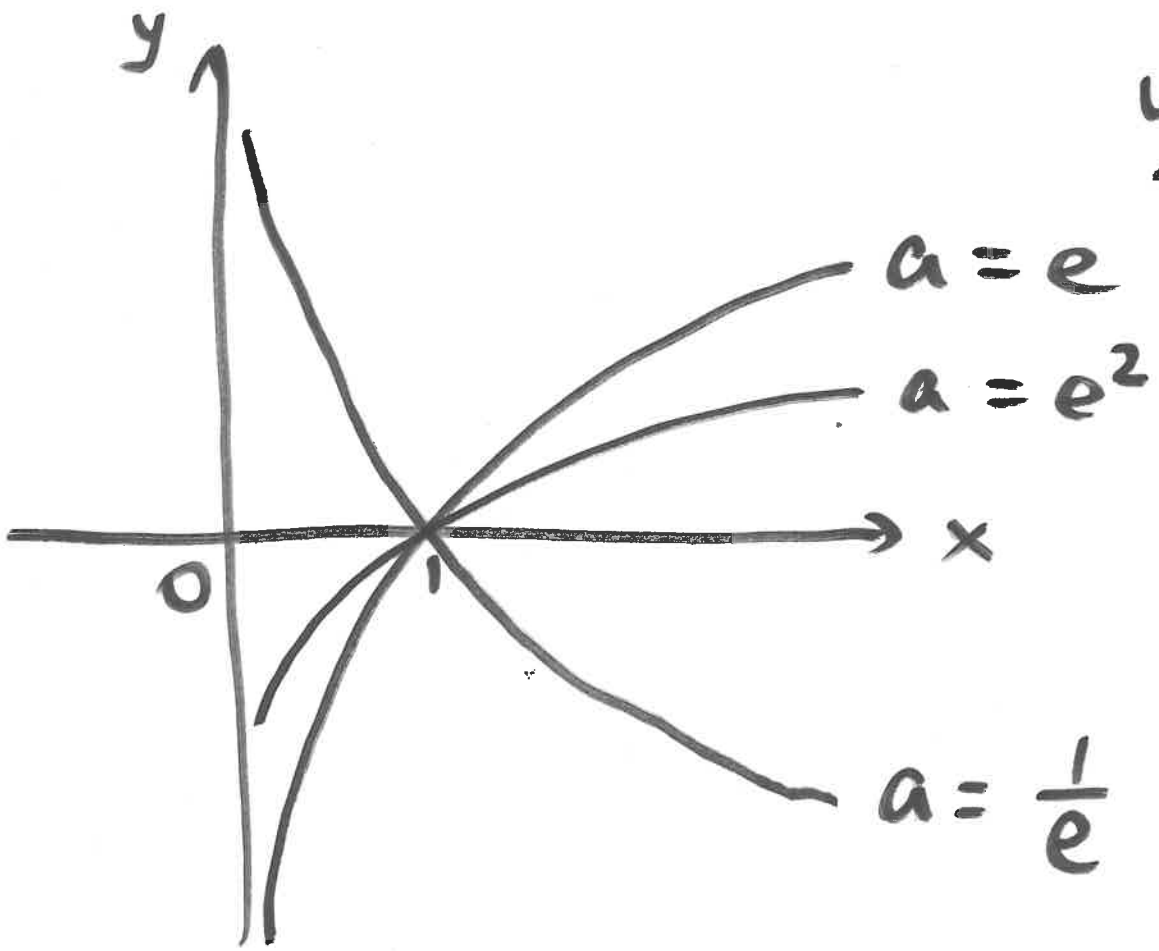
• Graph




$$y = a^x = e^{x \ln a}$$

$a > 1 \quad \ln a > 0$

$0 < a < 1 \quad \ln a < 0$




$$y = \log_a x = \frac{\ln x}{\ln a}$$

ex 1 $(9^{-2x^2})' = (e^{-2x^2 \ln 9})'$

$= e^{-2x^2 \ln 9} \cdot (-4x \ln 9)$

$(x^{2x})' = (e^{2x \ln x})'$

$= e^{2x \ln x} \cdot (2 \ln x + 2x \cdot \frac{1}{x})$

$= e^{2x \ln x} (2 \ln x + 2)$

$$\left((t^2+1)^{\sin t}\right)' = \left(e^{\sin t \cdot \ln(t^2+1)}\right)'$$

$$= e^{\sin t \cdot \ln(t^2+1)} \cdot \left(\cos t \cdot \ln(t^2+1) + \sin t \cdot \frac{1}{t^2+1} \cdot 2t\right)$$

$$\left(\log_{10} \sin x\right)' = \left(\frac{\ln(\sin x)}{\ln 10}\right)'$$

$$= \frac{1}{\ln 10} \cdot \frac{1}{\sin x} \cdot \cos x$$

ex 2 $\int 2^{\sin x} \cos x \, dx$

$= \int e^{\sin x \cdot \ln 2} \cos x \, dx$

$u = \sin x \cdot \ln 2$

$du = \cos x \cdot \ln 2 \cdot dx$

$= \frac{1}{\ln 2} \int e^u \, du$

$= \frac{1}{\ln 2} e^u + C = \frac{1}{\ln 2} e^{\sin x \cdot \ln 2} + C$

$$\int \frac{\log_4 x}{x}\,dx$$

$$= \int \frac{\ln x}{\ln 4 \cdot x}\,dx$$

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$= \frac{1}{\ln 4}\int u\,du$$

$$= \frac{1}{\ln 4}\cdot\frac{1}{2}u^2 + C = \frac{1}{2\ln 4}(\ln x)^2 + C$$

ex 3 Solve $2^{x^2} - 4^{2x} = 0$

$$2^{x^2} = 4^{2x}$$

$$\log_2(2^{x^2}) = \log_2(4^{2x})$$

$$x^2 = 2x \log_2 4 = 4x$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$x = 0$ , $x = 4$

**ex 4** Find the inverse of

$$f(x) = \ln\left(\frac{1}{1+2^x}\right)$$

$$y = \ln\left(\frac{1}{1+2^x}\right)$$

$$e^y = \frac{1}{1+2^x}$$

$$1+2^x = \frac{1}{e^y} = e^{-y}$$

$$2^x = e^{-y} - 1$$

$$x = \log_2(e^{-y} - 1)$$

$$f^{-1}(x) = \log_2(e^{-x} - 1)$$

ex 5 Solve $2^x - 2^{-x} = 3$

$y = 2^x$

$y - \frac{1}{y} = 3$

$y^2 - 1 = 3y$

$y^2 - 3y - 1 = 0$

$y = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-1)}}{2} = \frac{3 \pm \sqrt{13}}{2}$

$x = \log_2 \frac{3 + \sqrt{13}}{2}$

~~$x = \log_2 \frac{3 - \sqrt{13}}{2}$~~

$\frac{3 - \sqrt{13}}{2} < 0$