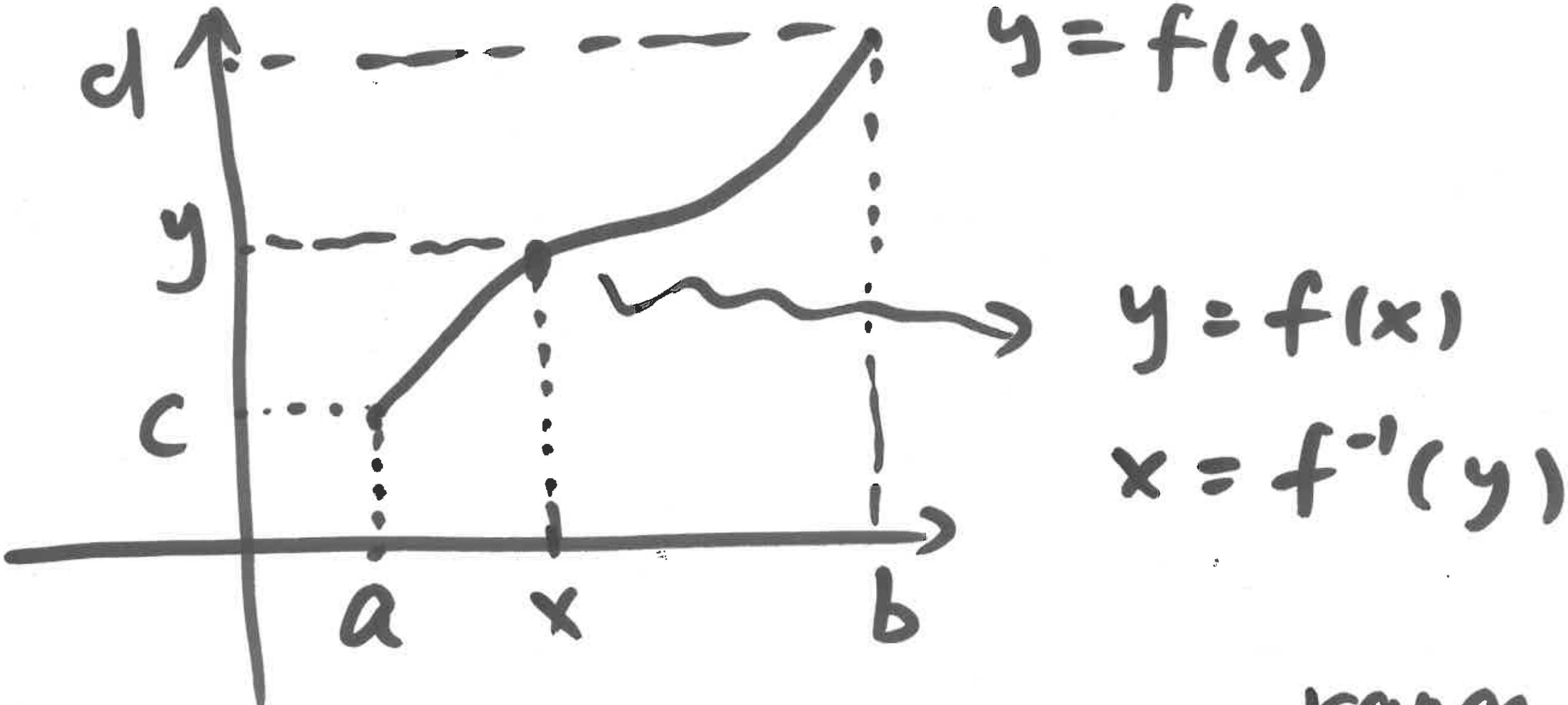
7.1 Inverse function

Def f is a function.

g is the inverse function of f (denoted by f^{-1}) if

$$f(x) = y \iff g(y) = x$$



domain of f: $[a, b]$: range of f^{-1}

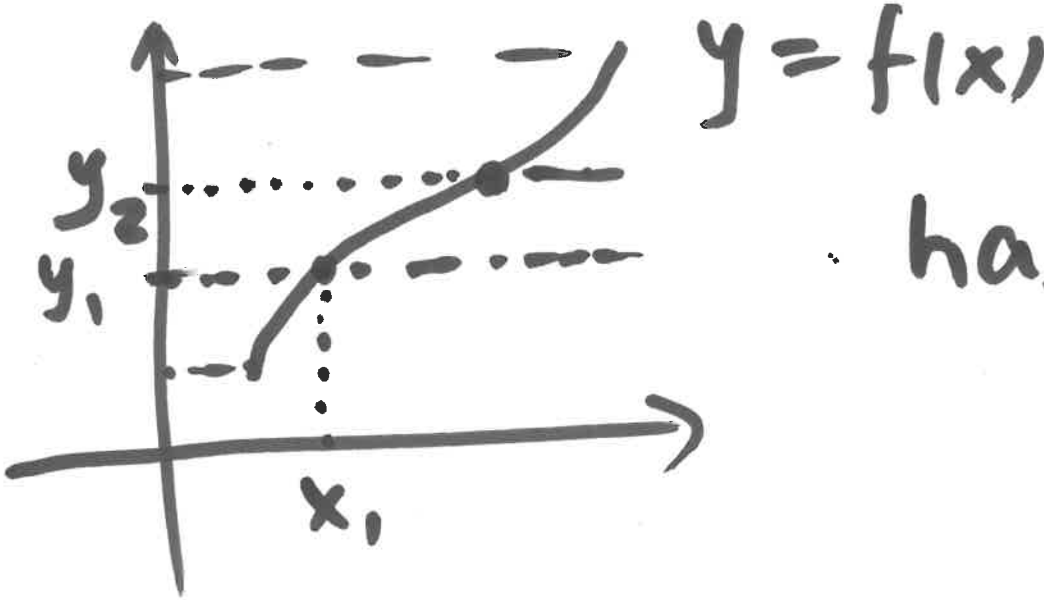
range of f: $[c, d]$: domain of f^{-1}

domain of f^{-1} = range of f

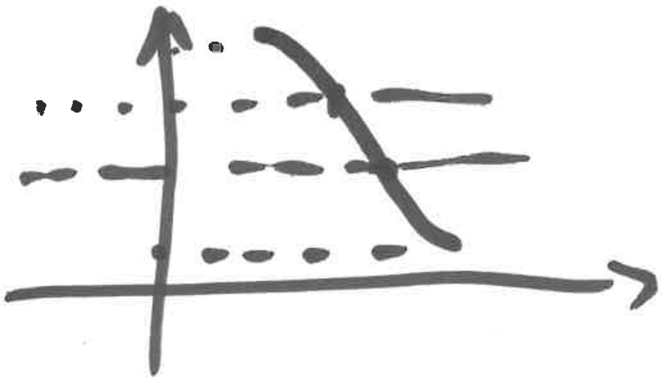
range of f^{-1} = domain of f.

- Determine whether f has an inverse:

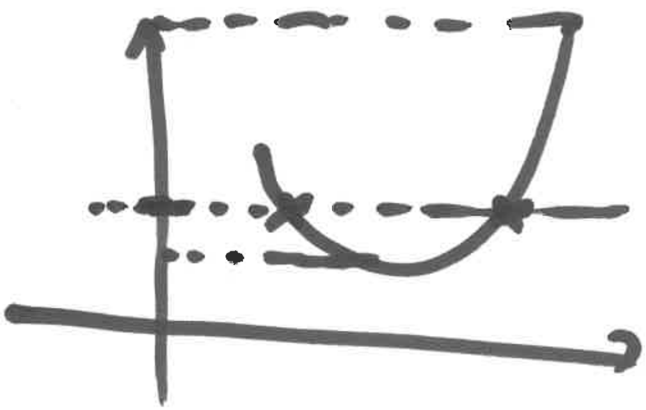
for any y in range of f, there is only one x with $y = f(x)$



has an inverse



has an inverse



no inverse

- Find the inverse of f (when f^{-1} exists)

$$y = f(x) \xrightarrow[\text{Solve for } x]{} x = \boxed{f^{-1}(y)}$$

when asked: $f^{-1}(x) = \ ?$
replace y by x

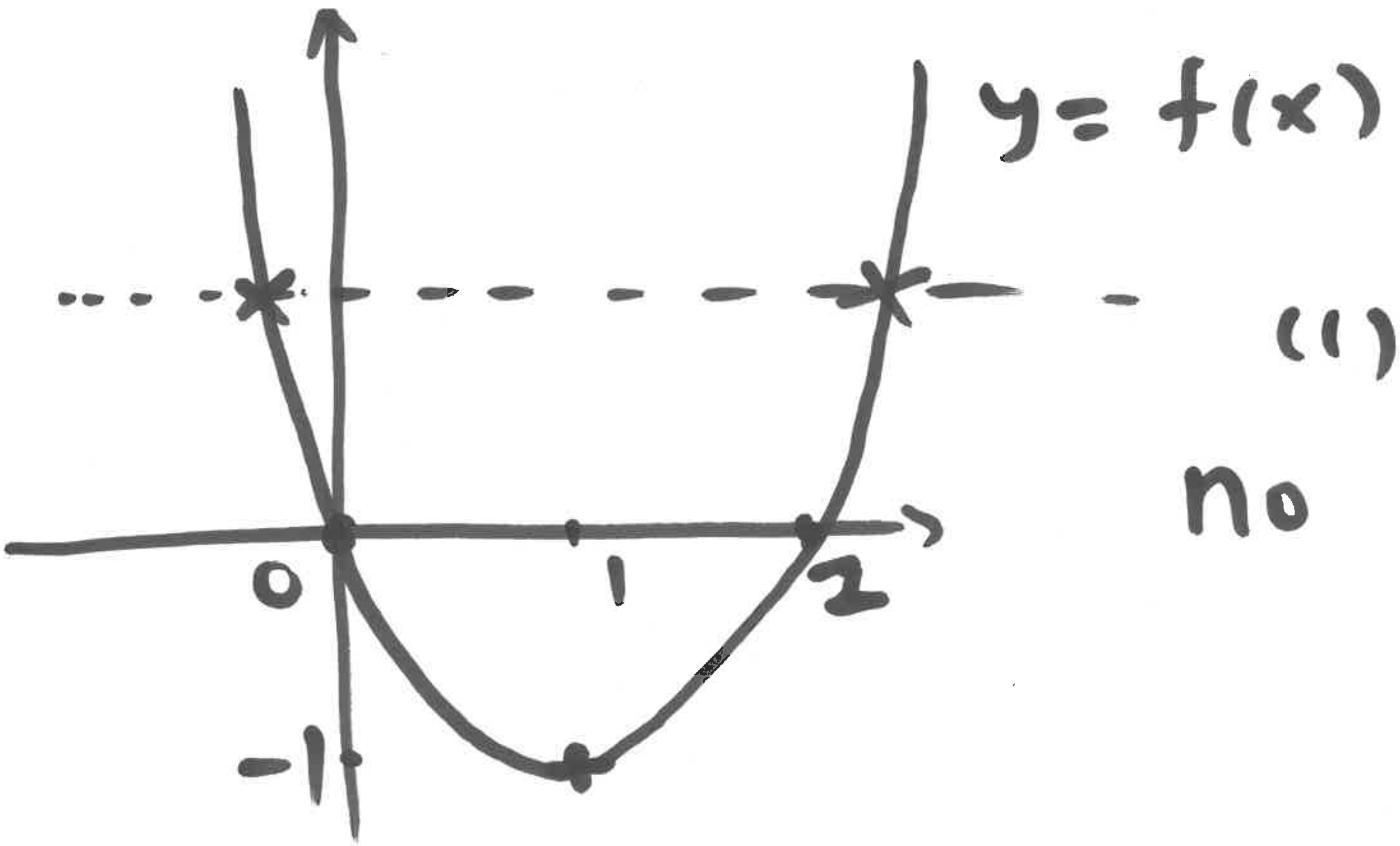
ex 1 Determine whether f has an inverse. If yes, find it.

(1) $f(x) = x^2 - 2x$, on $(-\infty, \infty)$

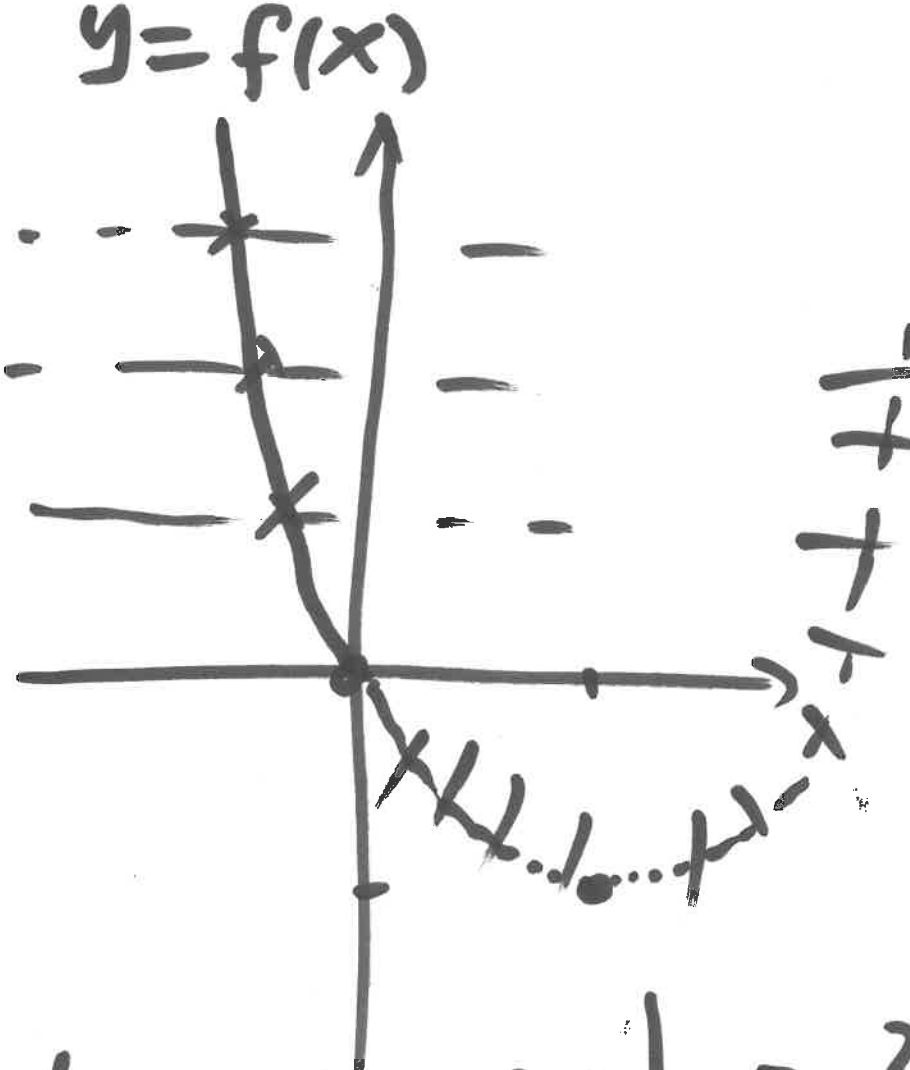
(2) .... $(-\infty, 0]$

(3) .... $[1, \infty)$

$f(x) = x^2 - 2x = (x-1)^2 - 1$



no inverse.



(2) has an inverse

$$y = x^2 - 2x$$

$$x^2 - 2x - y = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-y)}}{2}$$

$$= \frac{2 \pm \sqrt{4+4y}}{2}$$

"$x \le 0$"

$$= 1 \pm \sqrt{1+y} = 1 - \sqrt{1+y}$$

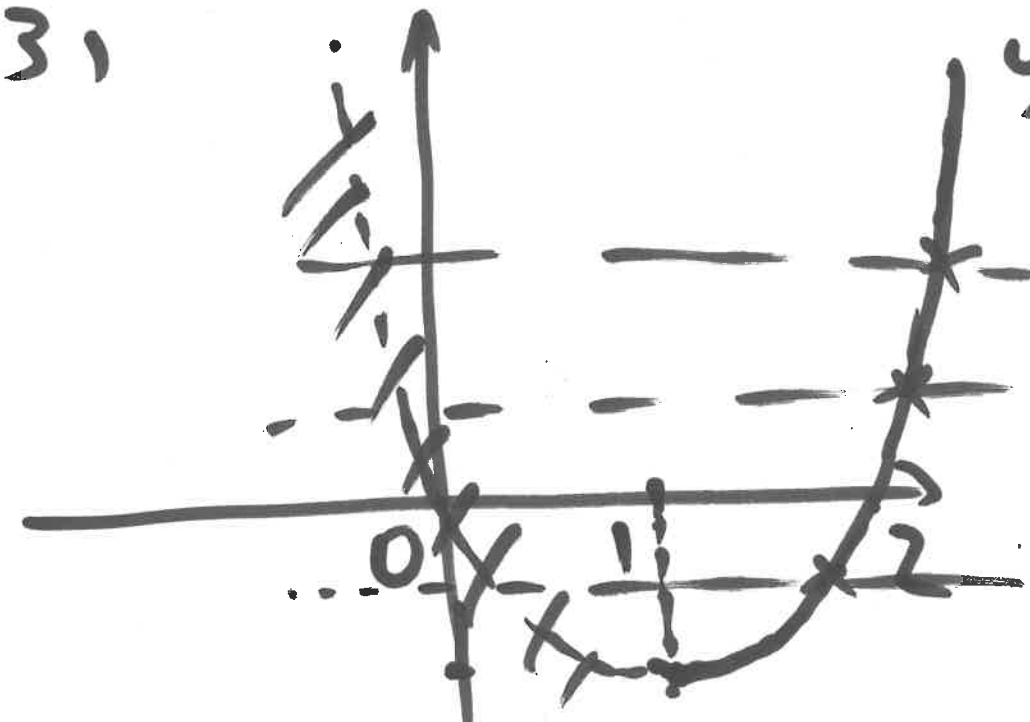
domain of f

$(-\infty, 0]$

$$f^{-1}(y) = 1 - \sqrt{1+y}$$

$$\boxed{f^{-1}(x) = 1 - \sqrt{1+x}}$$

(3)



$y = f(x)$ has an inverse

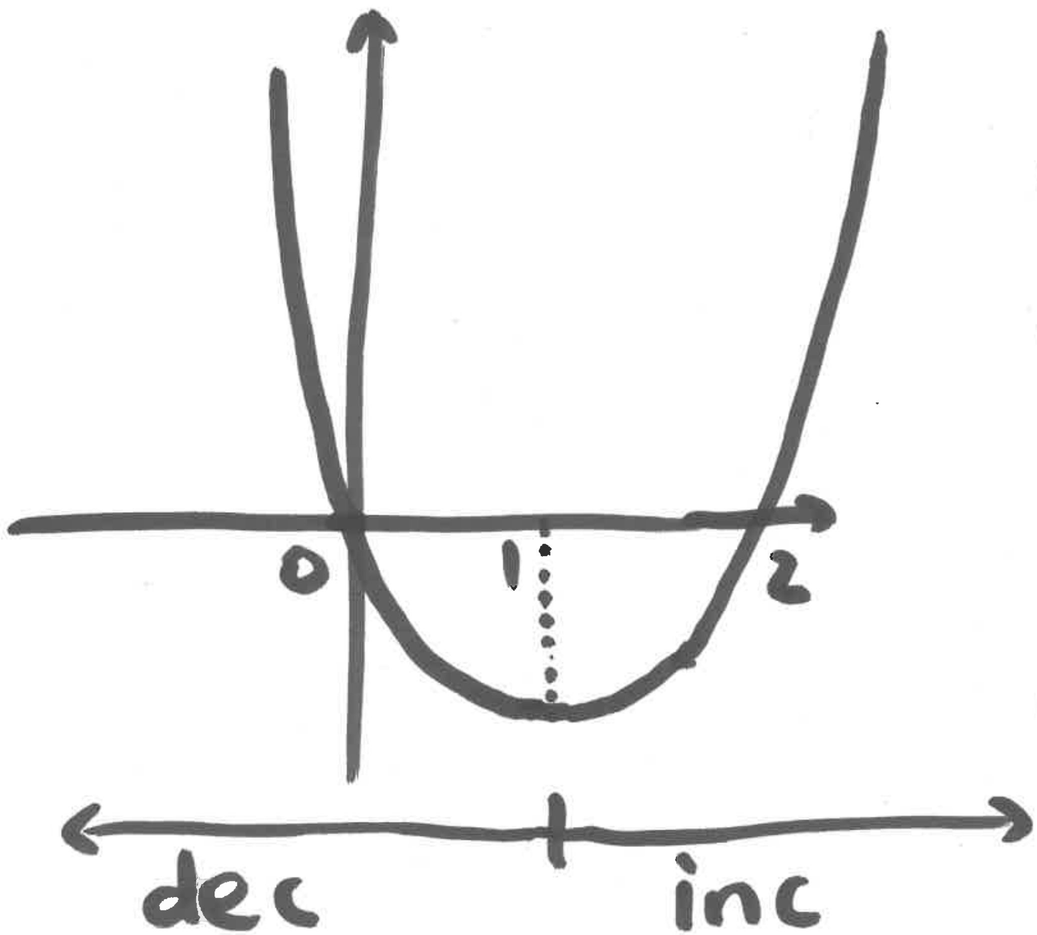
$$x = 1 \pm \sqrt{1+y}$$
$$= 1 + \sqrt{1+y}$$

Domain of f: $[1, \infty)$ "$x \geq 1$"

$$f^{-1}(y) = 1 + \sqrt{1+y}$$

$$\boxed{f^{-1}(x) = 1 + \sqrt{1+x}}$$

(4) What is the largest interval containing 0 on which $f(x) = x^2 - 2x$ has an inverse?

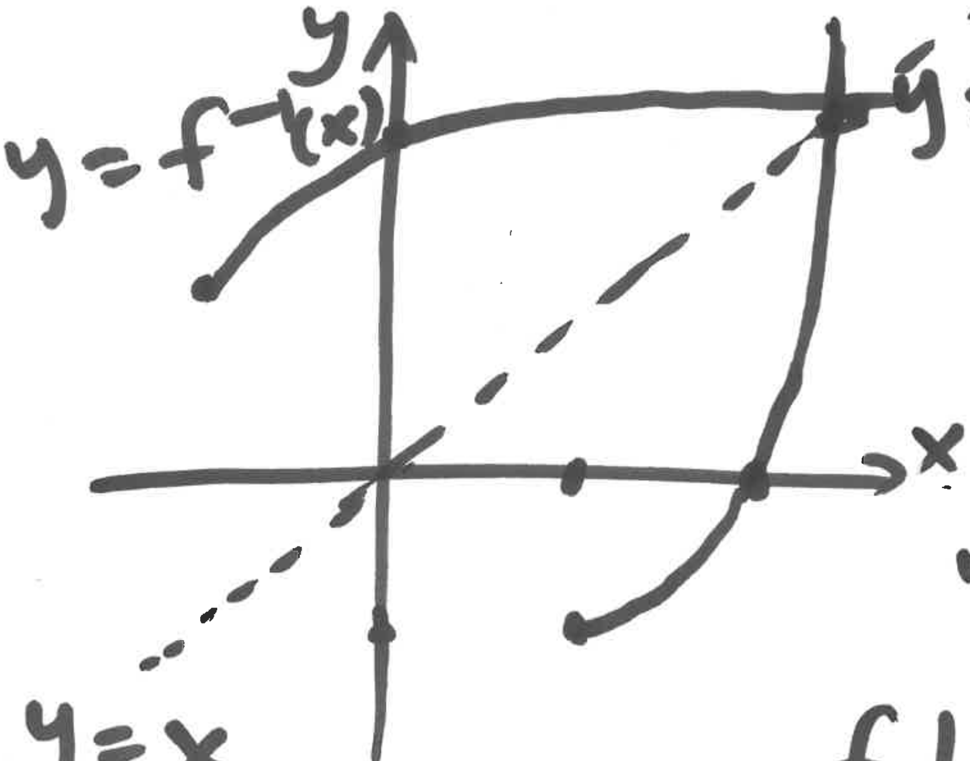


" the largest monotone interval containing $x = 0$ "

$\boxed{(-\infty, 1]}$

(decreasing interval)

• Graph of $y = f^{-1}(x)$



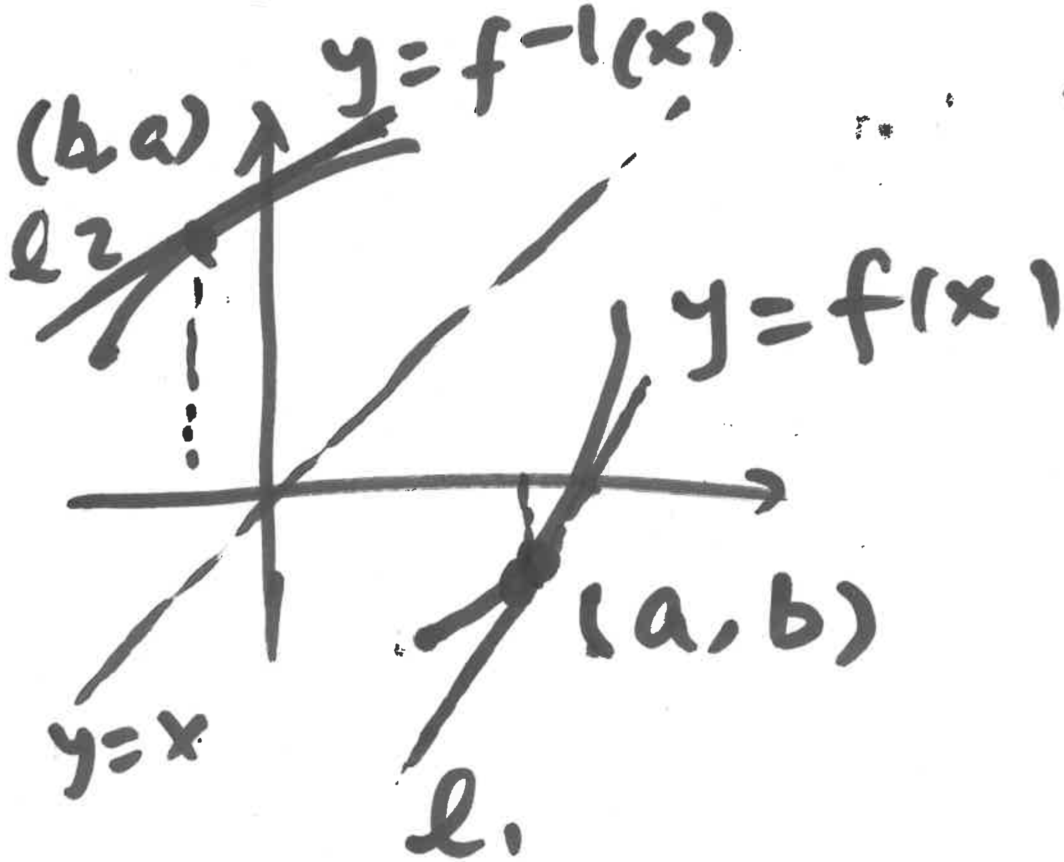
$y = f(x) \iff \underline{x = f^{-1}(y)}$

$\underline{y = f^{-1}(x)}$

"exchange x and y"

<u>flip through $y = x$</u>

- Derivative of $f^{-1}(x)$



$$\boxed{\text{slope of } \ell_1 \times \text{slope of } \ell_2 = 1}$$

$$f'(a)\cdot\left(f^{-1}\right)'(b)=1$$

$$\boxed{b=f(a) \Rightarrow \left(f^{-1}\right)'(b)=\frac{1}{f'(a)}}$$

ex 2 $\quad f(x) = x^2 - 2x$

on $[1, \infty)$

"b"

$(f^{-1})'(0) = ?$

$0 = f(x) \qquad x^2 - 2x = 0$

$x(x-2) = 0 \qquad x = 2$ "a"

$(f^{-1})'(0) = \frac{1}{f'(2)} = \frac{1}{2 \cdot 2 - 2} = \boxed{\frac{1}{2}}$

$f'(x) = 2x - 2$