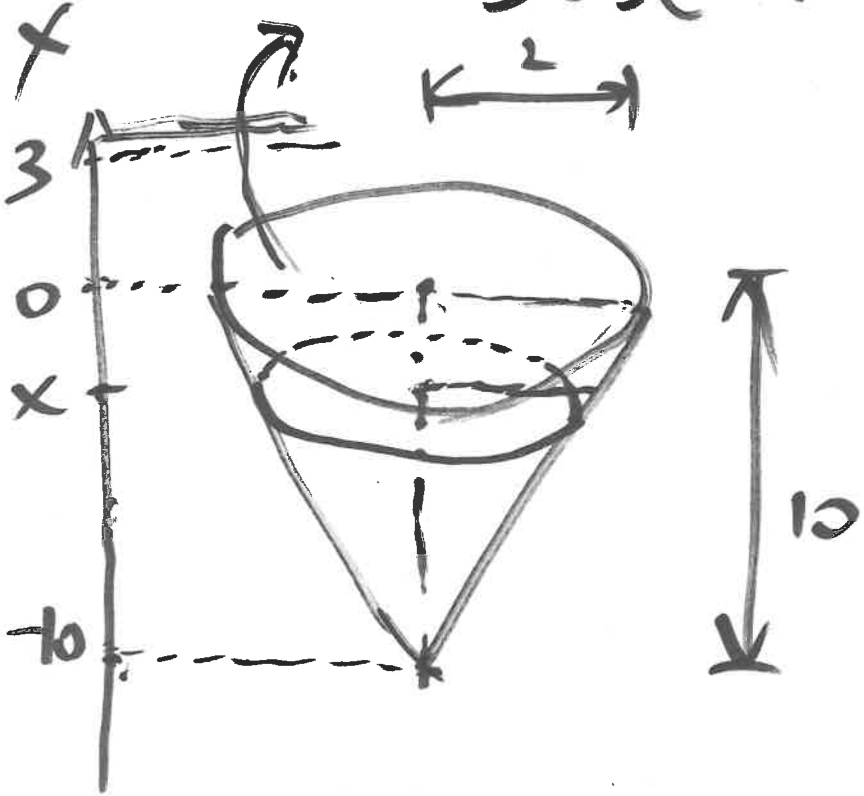
ex Cone shaped tank

height = 10 feet

base radius = 2 feet



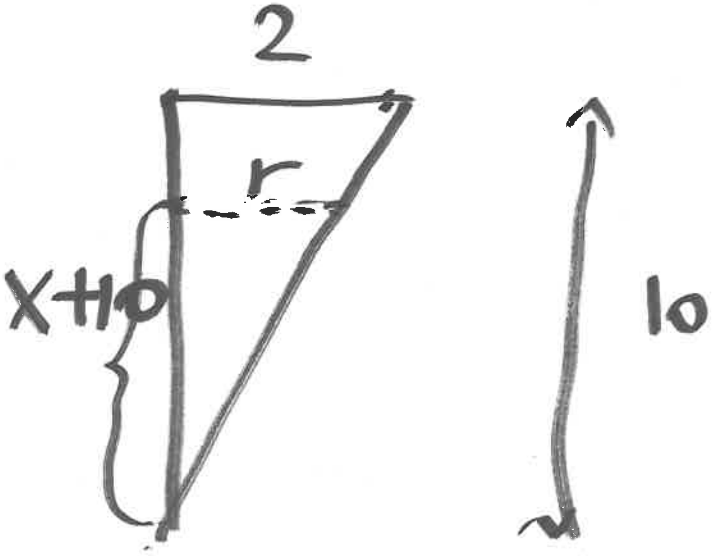
pump out water

to height = 3 feet

Work?

$a = -10 \quad b = 0$

$l = 3$



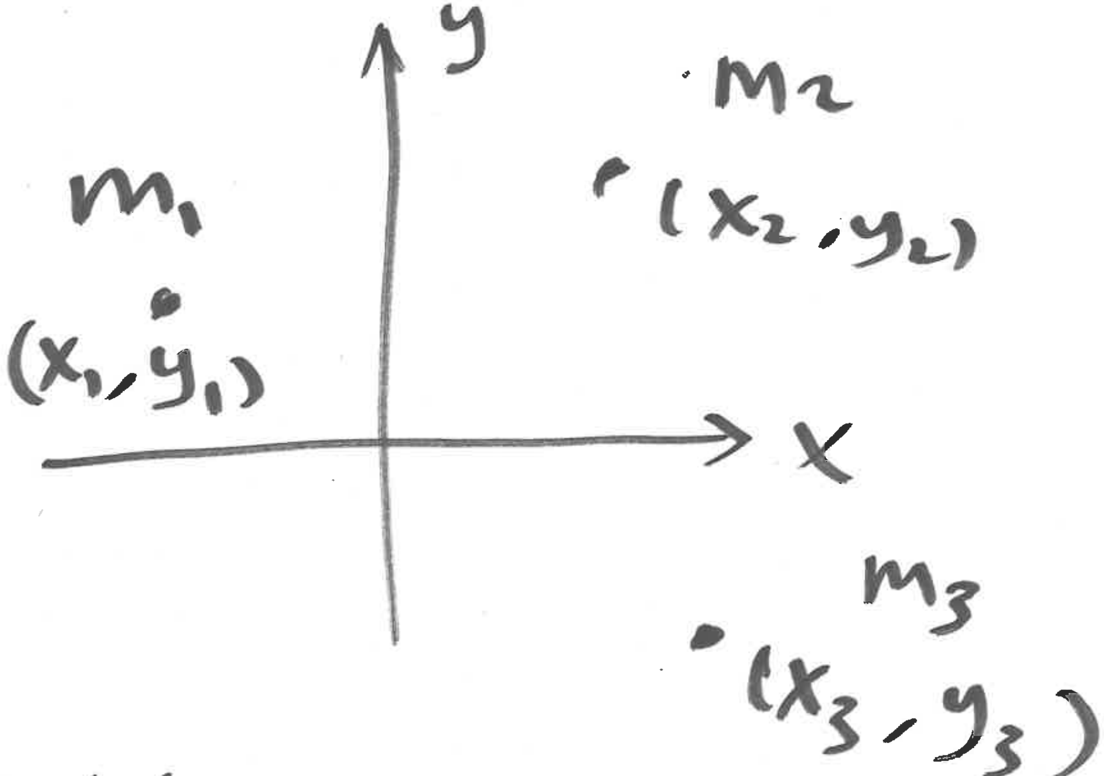
$$\frac{r}{x+10} = \frac{2}{10}$$

$$r = \frac{1}{5}(x+10)$$

$$= \frac{1}{5}x + 2.$$

$$W = \int_{-10}^{0} 62.5 \cdot \underbrace{\pi \left(\frac{1}{5}x + 2\right)^2}_{\text{area}} \cdot (3 - x)\, dx$$

6.5 Moments, center of gravity



N point masses

at (x_i, y_i)

w/ mass m_i

$i = 1, \ldots, N$

Moments: $M_y = \sum_{i=1}^{N} m_i x_i$

$$M_x = \sum_{i=1}^{N} m_i y_i$$

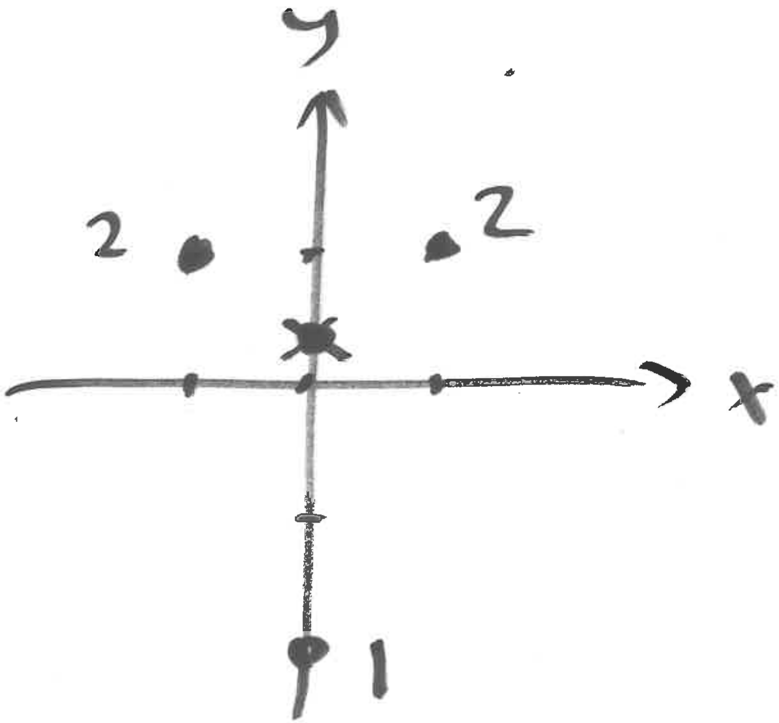
Center of gravity :

$$(\bar{x}, \bar{y}) \qquad \bar{x} = \frac{M_y}{m}, \quad \bar{y} = \frac{M_x}{m}$$

$$m = \sum_{i=1}^{N} m_i$$

$$\bar{x} = \frac{m_1 x_1 + \cdots + m_N x_N}{m_1 + \cdots + m_N}$$

ex 1 3 point masses w/ mass $2, 2, 1$, at $(-1,1)$, $(1,1)$, $(0,-2)$. Compute M_y, M_x, $\bar{x}$, $\bar{y}$.



$$M_y = 2\cdot(-1) + 2\cdot 1 + 1\cdot 0 = 0.$$

$$M_x = 2\cdot 1 + 2\cdot 1 + 1\cdot(-2) = 2$$

$$m = 2 + 2 + 1 = 5$$

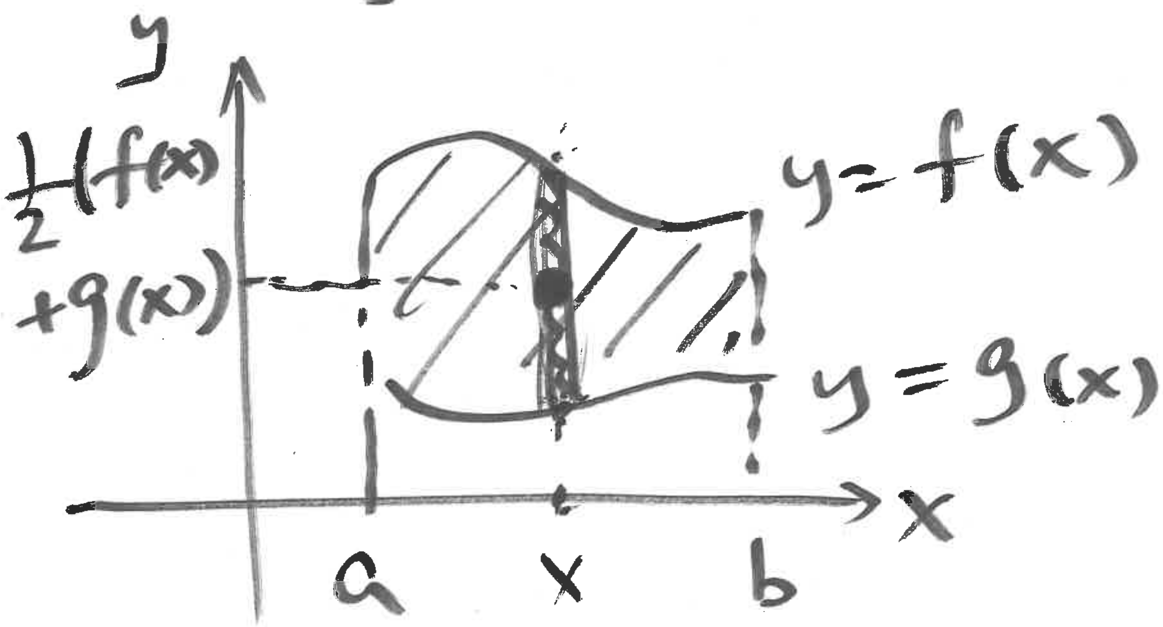
$$\bar{x} = \frac{0}{5} = 0$$

$$\bar{y} = \frac{2}{5}$$

center of gravity

$$\left(0, \frac{2}{5}\right)$$

- M_y for plane region



$$\boxed{M_y = \int_a^b \underset{\uparrow\ \text{"}x_i\text{"}}{x}\, \underbrace{(f(x)-g(x))\,dx}_{\text{"}m_i\text{"}}}$$

$$M_x = \int_a^b \underbrace{\frac{1}{2}\left(f(x)+g(x)\right)}_{\text{"}y_i\text{"}} \underbrace{\left(f(x)-g(x)\right)}_{\text{"}m_i\text{"}} dx$$

$$\boxed{M_x = \int_a^b \frac{1}{2}\left(f(x)^2 - g(x)^2\right) dx}$$

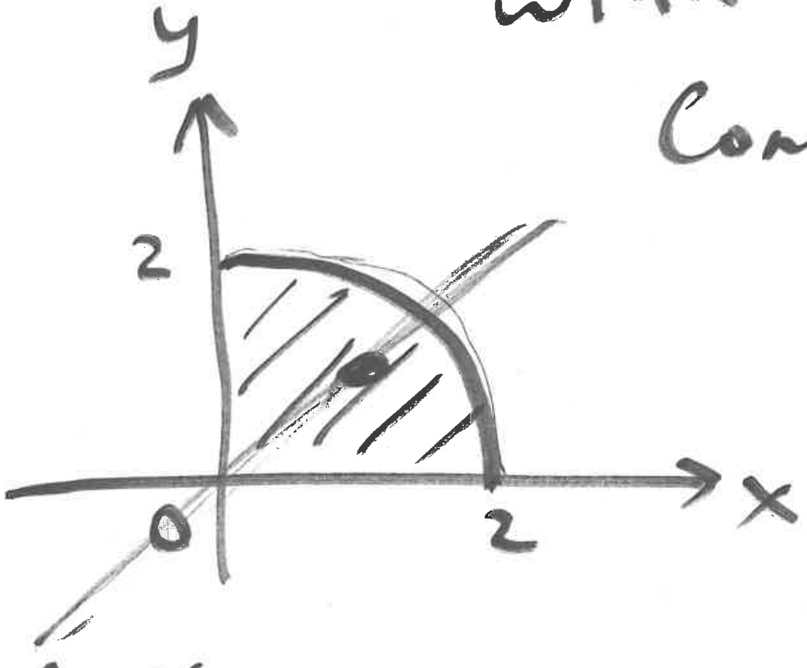
center of gravity

$$\bar{x} = \frac{M_y}{A} \qquad A = \int_a^b (f(x) - g(x))\,dx$$

$$\bar{y} = \frac{M_x}{A}$$

ex 2 Quarter of a circle with radius = 2

Compute M_y, M_x, $\bar{x}$, $\bar{y}$.



Circle: $x^2+y^2=4$

$y=\sqrt{4-x^2} \leftarrow f(x)$.

$g(x)=0$

$\bar{x}=\bar{y}$.

$\Rightarrow M_x = M_y$

$$\boxed{M_x = \frac{8}{3}}$$

$$M_y = \int_0^2 x\sqrt{4-x^2}\,dx = -\frac{1}{3}(4-x^2)^{3/2}\Big|_0^2$$

$$= \frac{8}{3}.$$

$$\int x\sqrt{4-x^2}\,dx$$

$$u = 4 - x^2 \qquad du = -2x\,dx$$

$$= -\frac{1}{2}\int u^{1/2}\,du$$

$$= -\frac{1}{3}u^{3/2} + C.$$