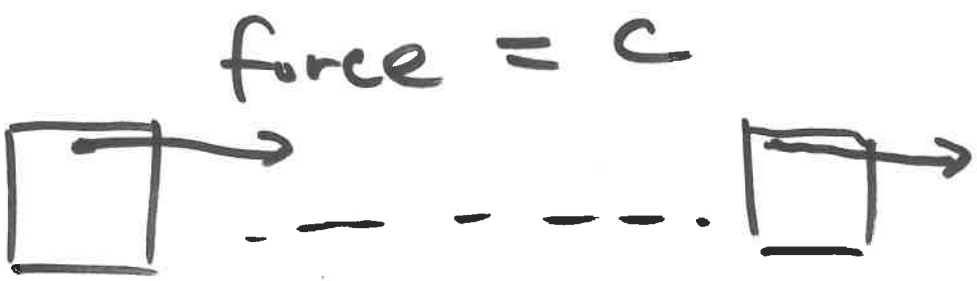
6.4 Work

- Constant force

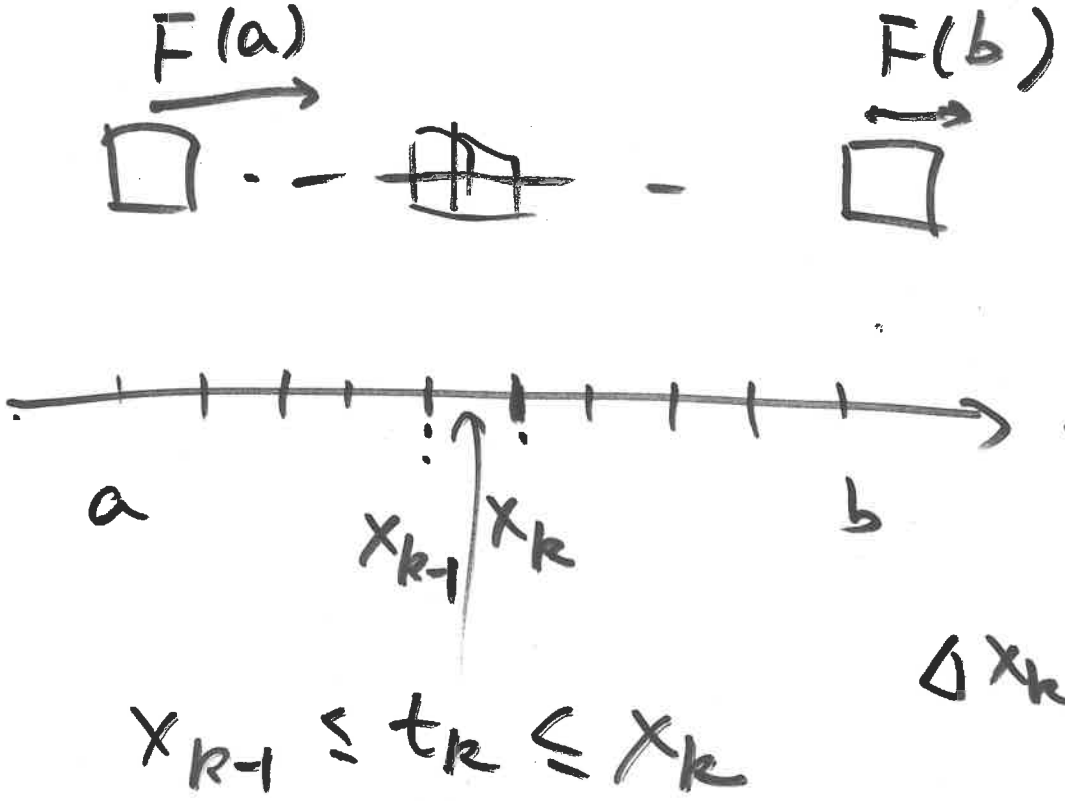


Work

$W = c \cdot (b-a)$.

- Non-constant force

force = $F(x)$.



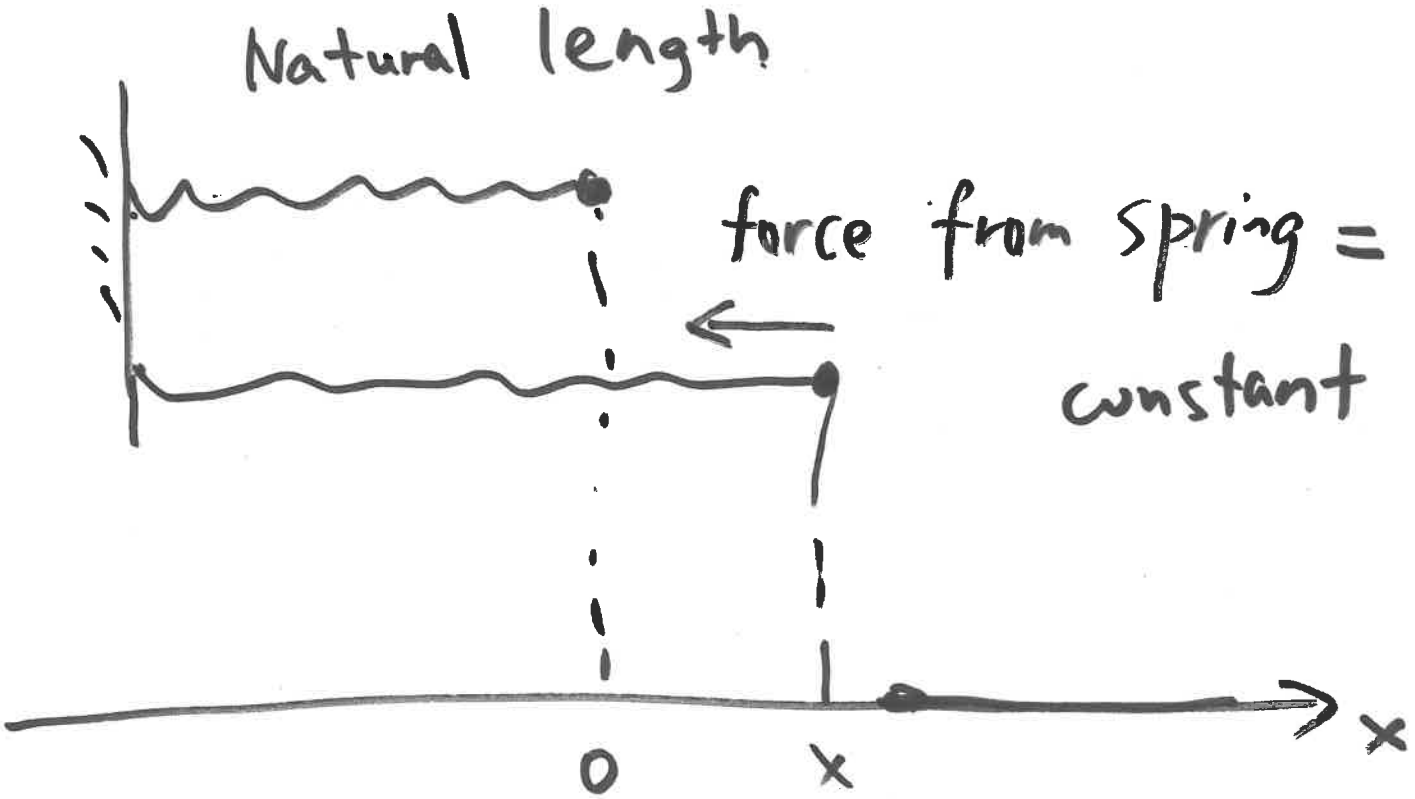
$$W \approx \sum_k F(t_k)\,\Delta x_k$$

$$\boxed{W = \int_a^b F(x)\,dx}$$

$$\Delta x_k = x_k - x_{k-1}$$

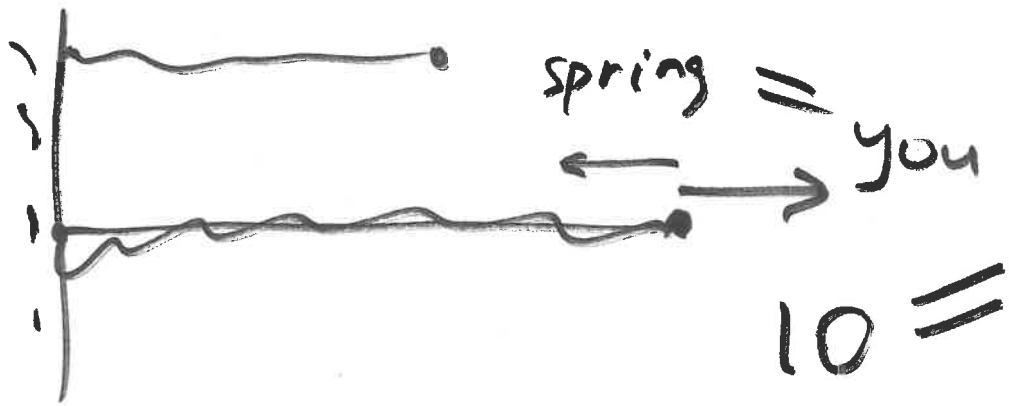
$$x_{k-1} \le t_k \le x_k$$

ex 1 Springs satisfy Hooke's Law



Suppose the work to stretch a spring from $x = 1$ to $x = 2$ is 10.

What is work to stretch from $x = 2$ to $x = 3$?



$$F(x) = kx$$

$$10 = W_{1 \to 2} = \int_1^2 kx \, dx$$

$$10 = \int_1^2 kx\,dx = k\int_1^2 x\,dx$$

$$= k\cdot\frac{1}{2}x^2\Big|_{x=1}^{2} = \frac{3}{2}k$$

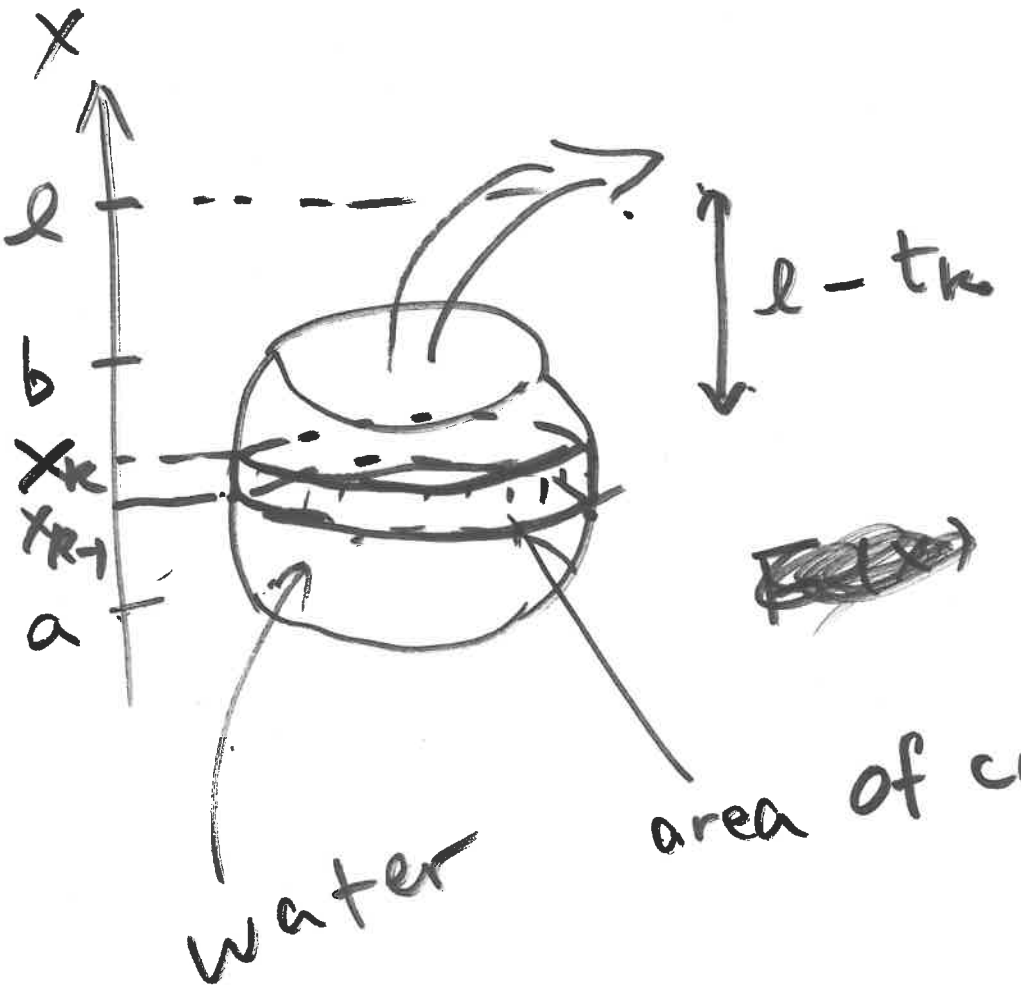
$$k = \frac{2}{3}\cdot 10 = \frac{20}{3}.$$

$$W_{2\to 3} = \int_2^3 kx\,dx = \frac{20}{3}\int_2^3 x\,dx$$

$$= \frac{20}{3}\,\frac{1}{2}x^2\Big|_2^3 = \frac{20}{3}\cdot\frac{1}{2}\cdot 5 = \boxed{\frac{50}{3}}$$

Pump water out of tank

density.



$$F_{piece} = \rho \cdot A(t_k) \cdot \Delta x_k$$

$$Work_{piece} = F_{piece}(\ell - t_k)$$

$$W \approx \sum_k \rho \cdot A(t_k) \cdot (\ell - t_k) \cdot \Delta x_k$$

$$\boxed{W = \int_a^b \rho \cdot A(x)(\ell - x)\, dx}.$$

ρ for water $= 62.5$ pound/foot^3

pound, foot