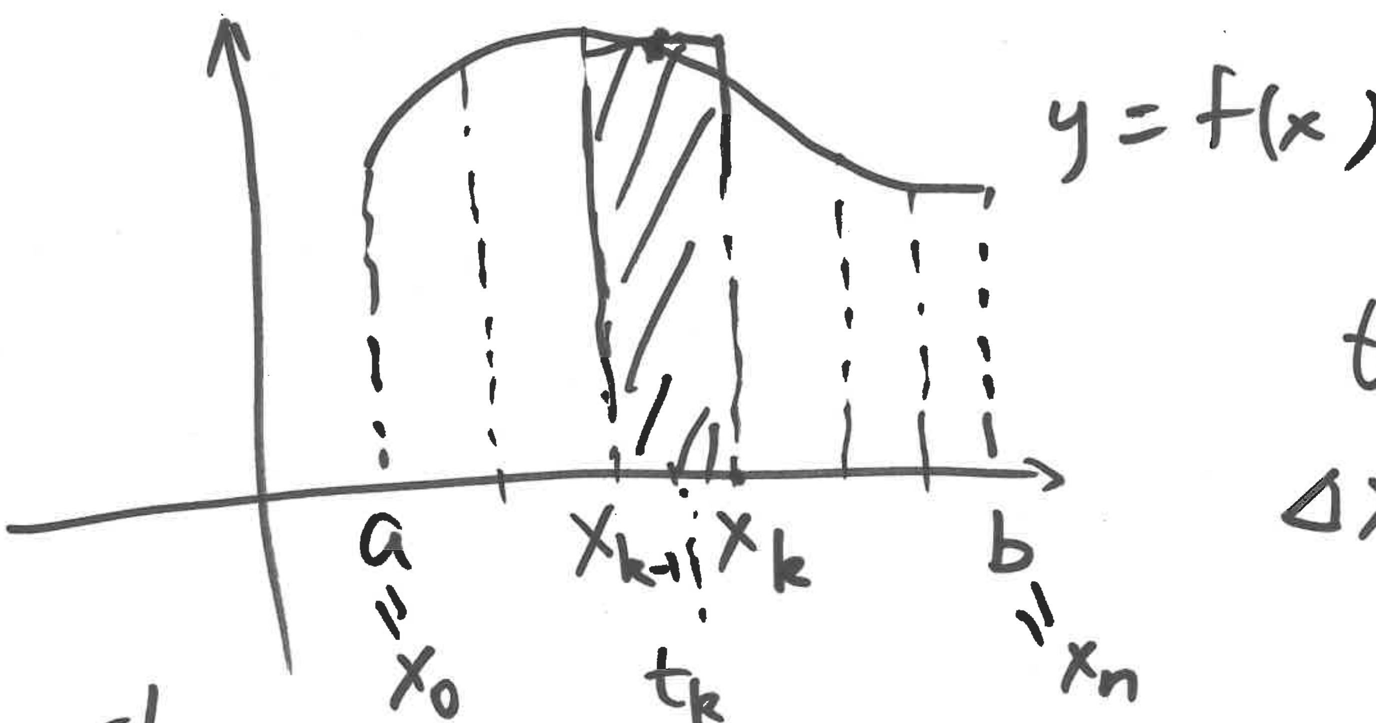
6.1 Volume

- Riemann sums



$t_k \in [x_{k-1}, x_k]$

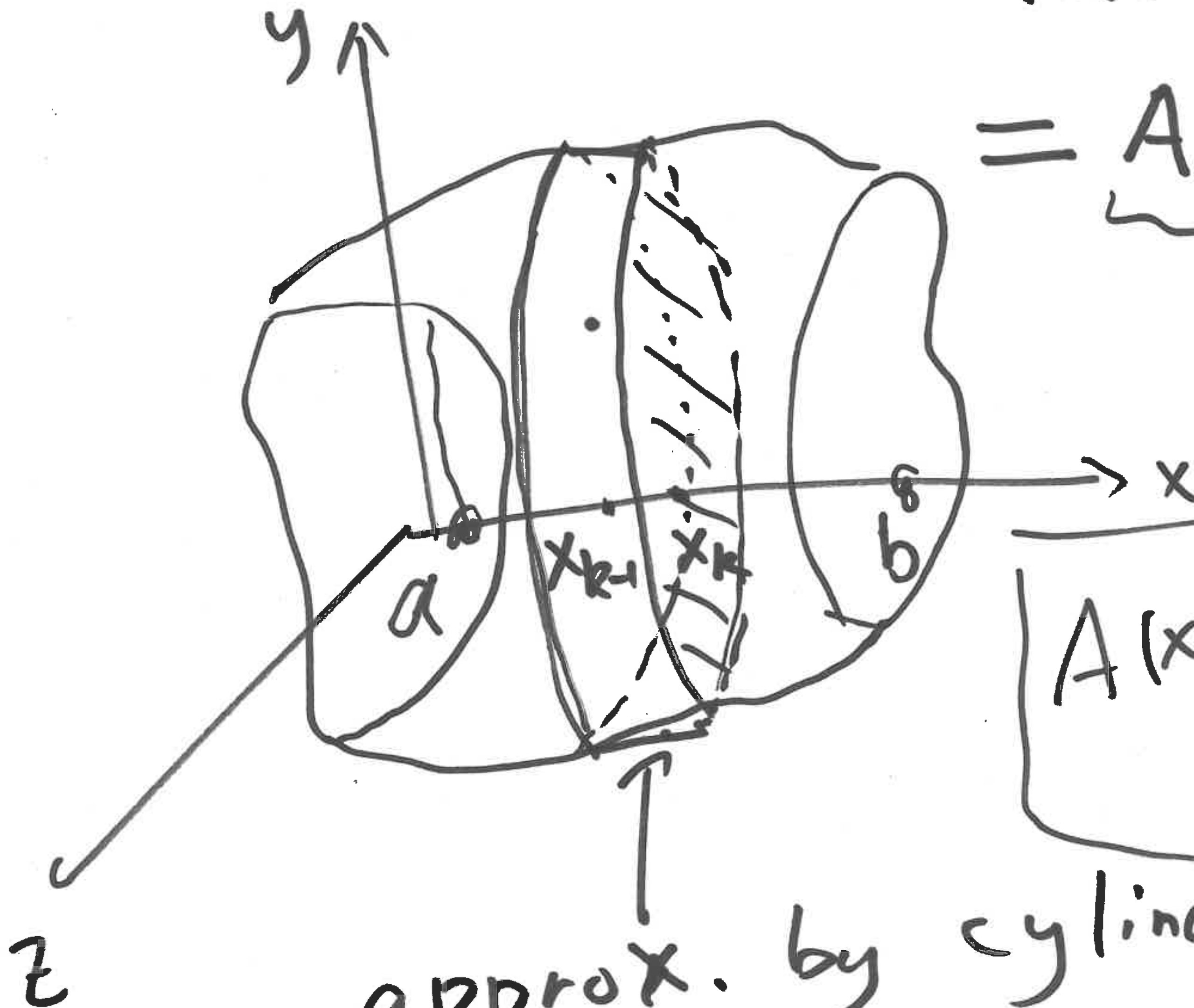
$\Delta x_k = x_k - x_{k-1}$

$$\int_a^b f(x)\,dx \approx \sum_{k=1}^{n} f(t_k)\cdot \Delta x_k$$

Volume

volume of cylinder

$= \underbrace{A(x_k)}\, \Delta x_k$.

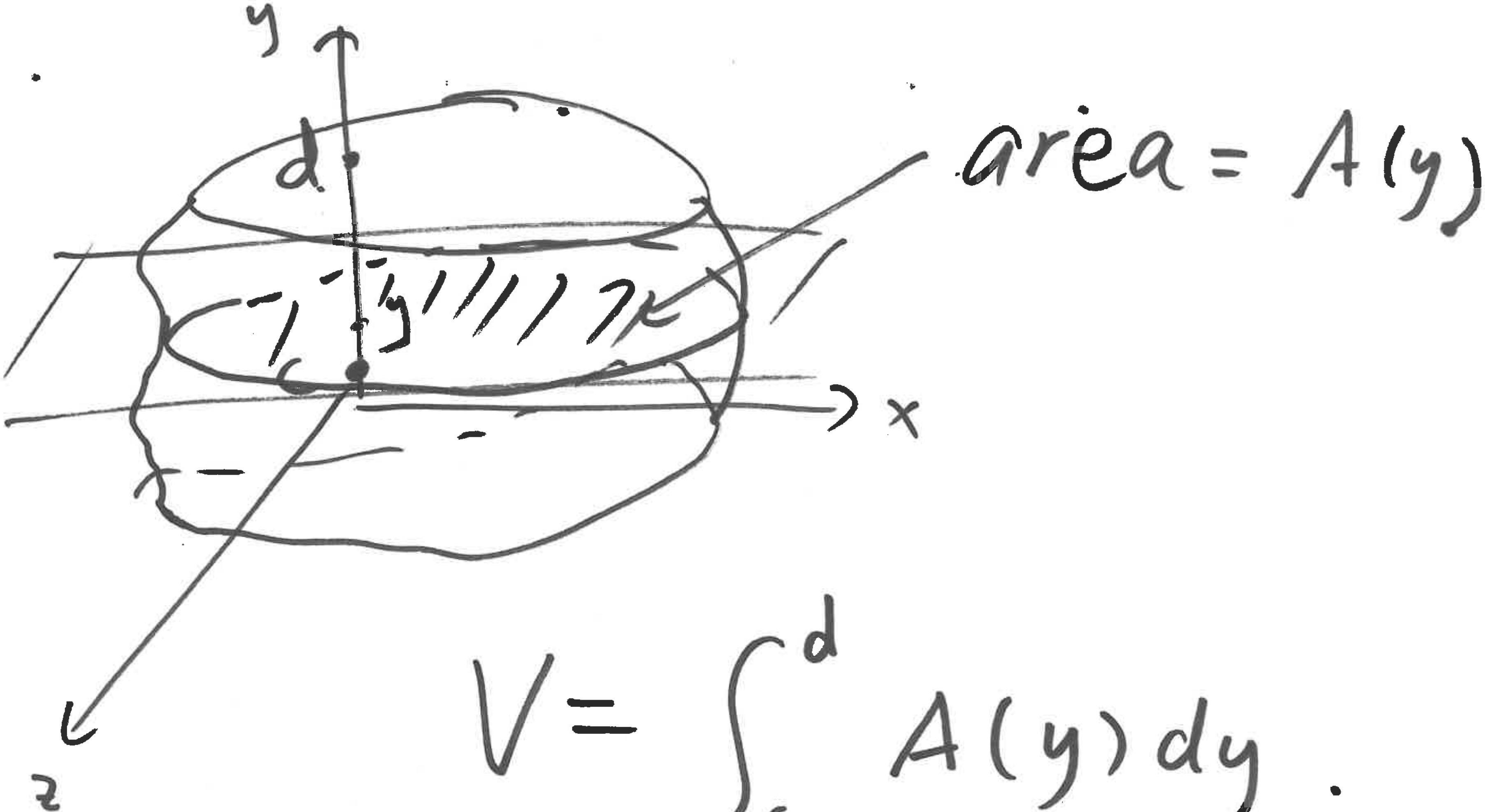


approx. by cylinder

$A(x)$: area of cross-section at x

Volume $V \approx \sum_{k=1}^{n} A(x_k)\,\Delta x_k$

$$\boxed{V = \int_a^b A(x)\,dx}$$

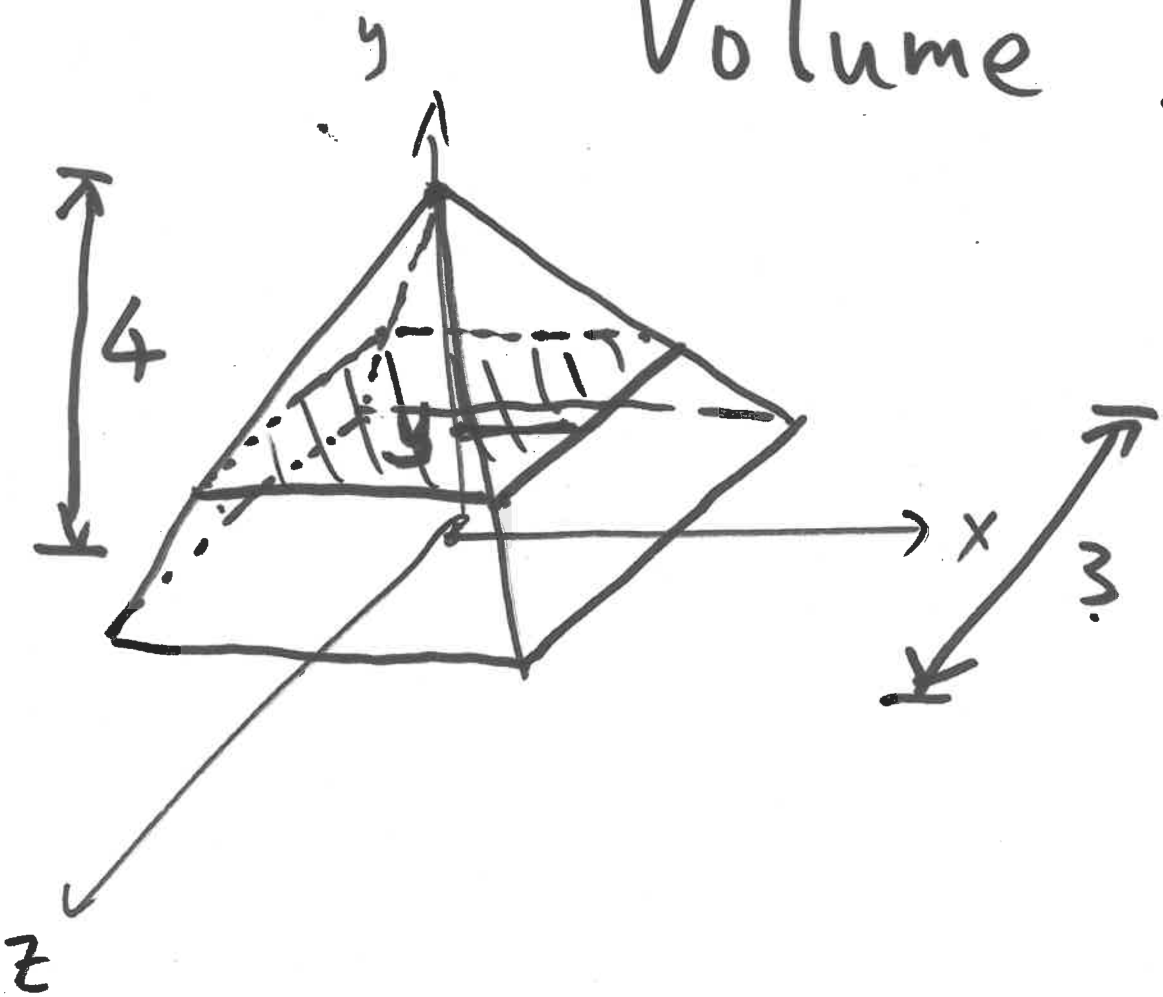


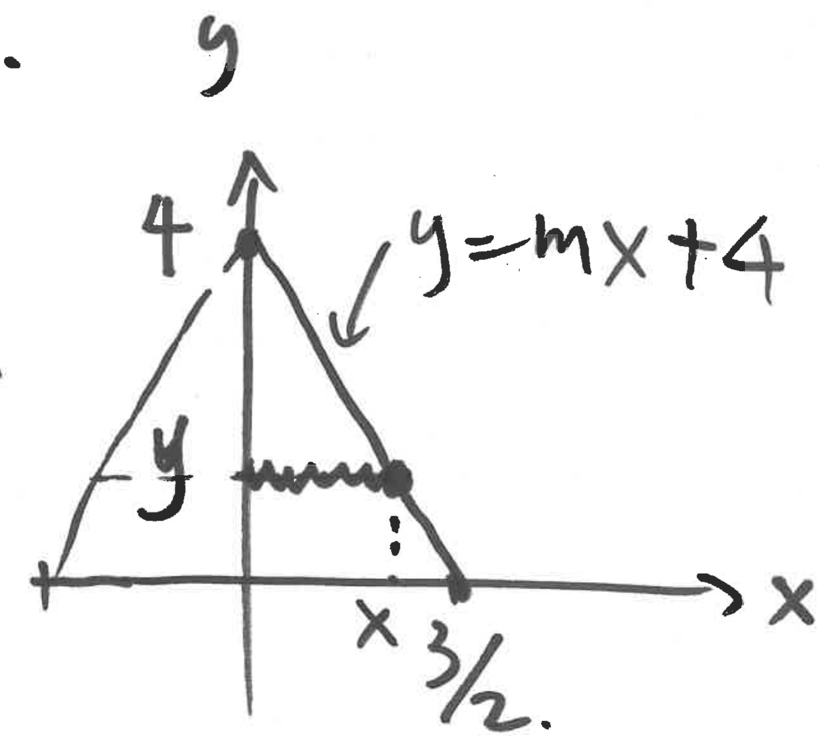
$$V = \int_c^d A(y)\,dy.$$

ex 1 Pyramid: tall = 4

side of basis = 3.

Volume?





$$y = mx + 4$$

$$0 = \frac{3}{2}m + 4 \qquad \frac{3}{2}m = -4$$

$$m = -\frac{8}{3}.$$

$$y = -\frac{8}{3}x + 4 \implies -\frac{8}{3}x = y - 4$$

$$x = -\frac{3}{8}y + \frac{3}{2}.$$

$$A(y) = \left[2\cdot\left(-\frac{3}{8}y + \frac{3}{2}\right)\right]^2.$$

$$= \left(-\frac{3}{4}y + 3\right)^2$$

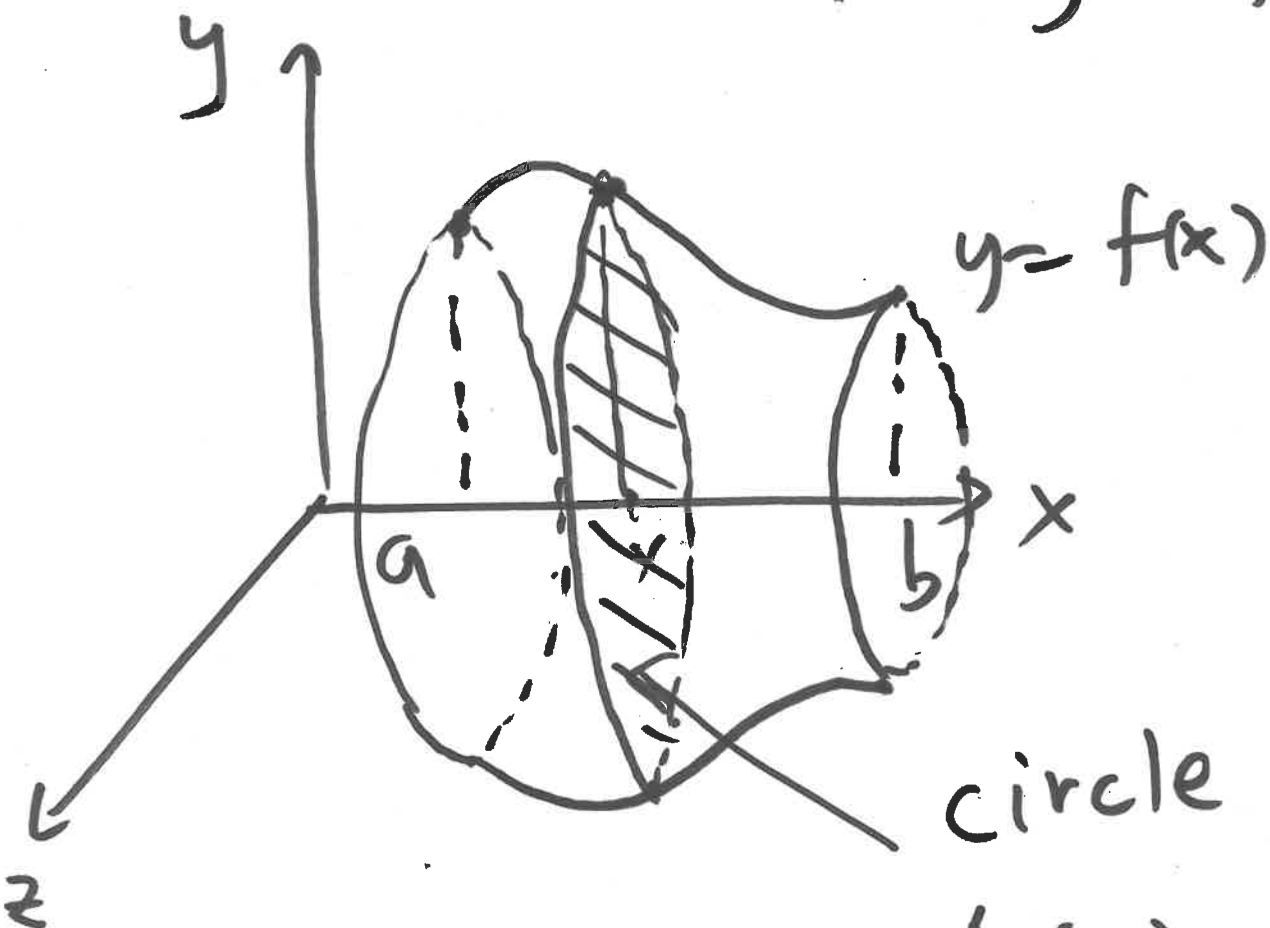
$$V = \int_0^4 \left(-\frac{3}{4}y + 3\right)^2 dy$$

$$= \int_0^4 \left(\frac{9}{16}y^2 - 2\cdot\frac{3}{4}\cdot 3y + 9\right) dy$$

$$= \left(\frac{3}{16}y^3 - \frac{9}{4}y^2 + 9y\right)\Big|_{y=0}^{4}$$

$$= \boxed{12}$$

Volume of solid by revolving the graph of $y = f(x)$.

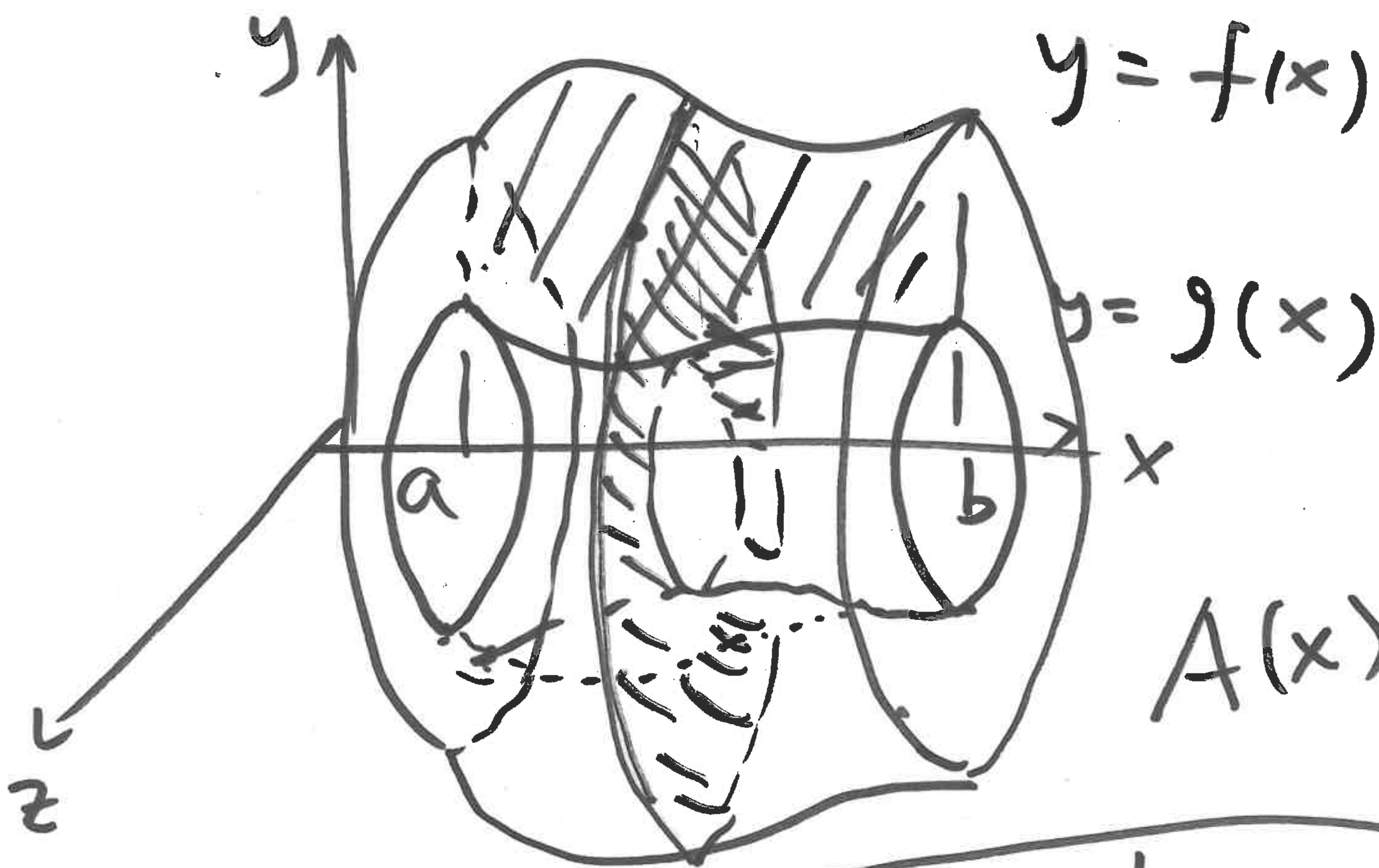


revolve around x-axis.

circle with radius $f(x)$

$$A(x) = \pi f(x)^2$$

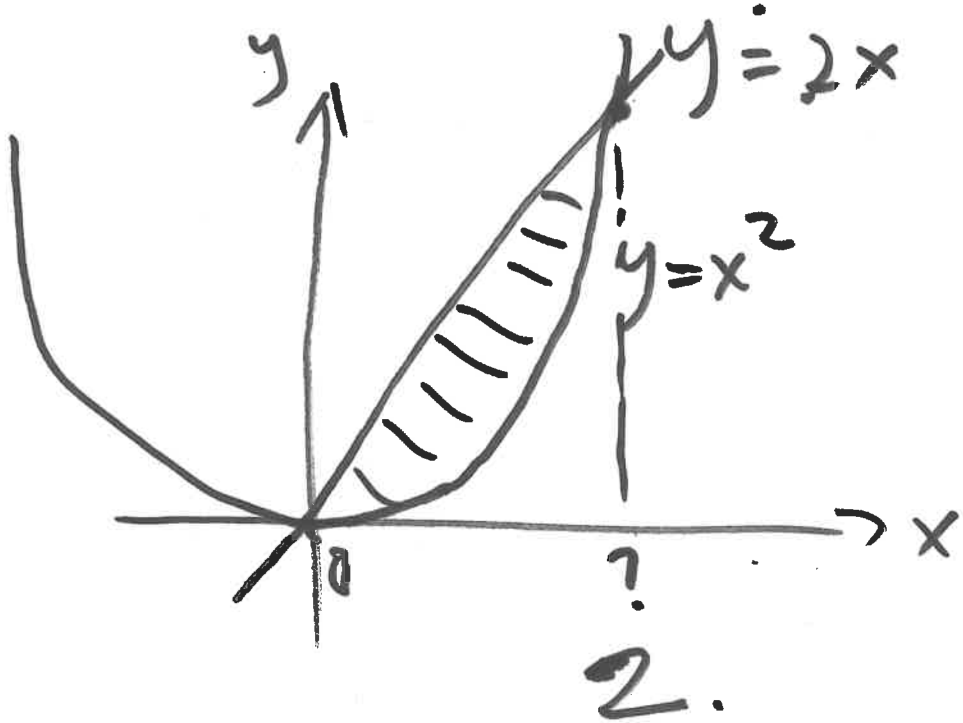
$$\boxed{V = \int_a^b \pi f(x)^2 \, dx}$$



$$A(x) = \pi f(x)^2 - \pi g(x)^2$$

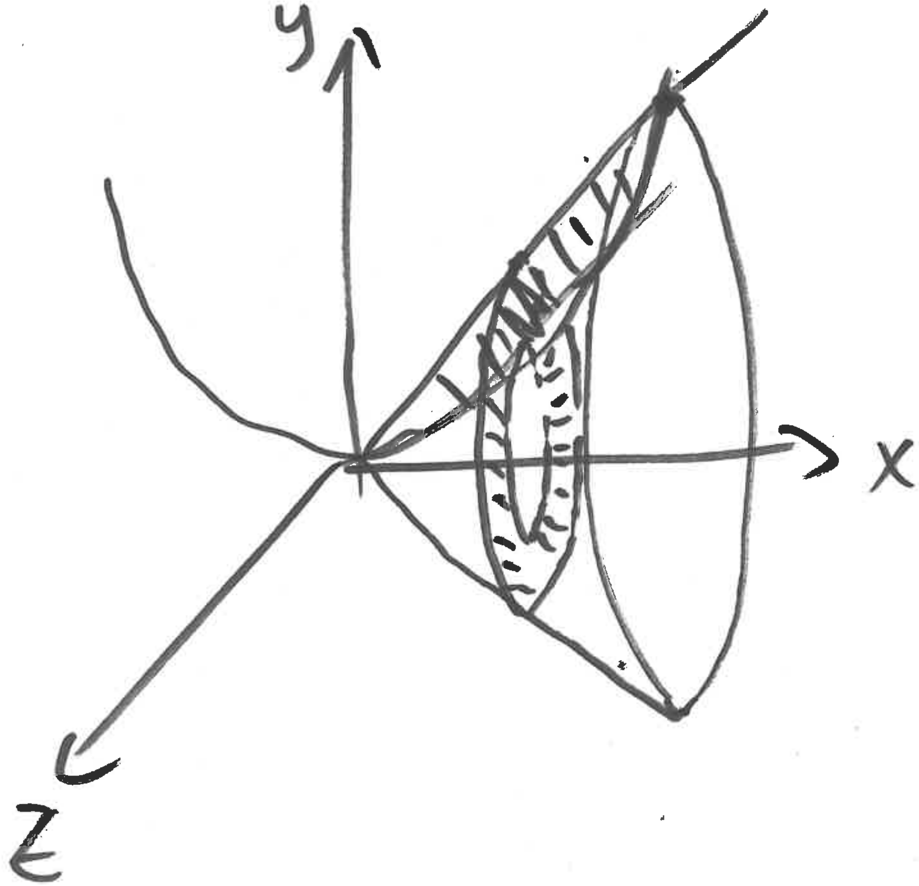
$$V = \int_a^b \pi \left(f(x)^2 - g(x)^2 \right) dx$$

ex 2 Region bounded by $y = 2x$ and $y = x^2$. What is the volume by revolving it around x-axis?



$f(x) = 2x$

$g(x) = x^2$


y
x
z

$$\begin{cases} y = 2x \\ y = x^2 \end{cases}$$

$$x^2 = 2x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$x = 0$ or $x = 2.$

$$V = \int_0^2 \pi \left((2x)^2 - (x^2)^2\right) dx.$$

$$= \pi \int_0^2 (4x^2 - x^4) dx$$

$$= \pi \left(4 \cdot \frac{1}{3}x^3 - \frac{1}{5}x^5\right)\Big|_0^2$$

$$= \pi \left(\frac{4}{3} \cdot 8 - \frac{1}{5} \cdot 2^5\right)$$

$$= \pi \cdot \frac{64}{15}.$$